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Viscous low Froude number flow interacting with mesoscale orography

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ABSTRACT

The interactions of viscous flow and orography are analysed with particular attention to the generation of vorticity and the shedding of vortices. It is found that at low levels in the flow (one fifth of the orography's height) no vortex shedding takes place, whereas at higher levels such as one-half, and three quarters of the orography's height vortex streets are formed. These simulations are performed by means of a three-dimensional, non-hydrostatic, non-linear numerical model. The flow investigated is continuously stratified.

1. Introduction

Studies on the interaction between a boundary layer and orography have been mainly restricted to small topographic features such as hills, and analytical theories describing the flow past such objects have been created. These theories are in most cases restricted to two dimensions and use linear simplifications.

The need to investigate the behaviour of boundary layer flow past isolated obstacles is motivated by the lack of knowledge in this area and its great practical importance because the portion of the atmosphere that interacts with the

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planet's surface is characterized by a high degree of turbulence. On the other hand, most of the research performed on the generation of vorticity and potential vorticity of flows past isolated mountains has been restricted to inviscid situations.

Amongst those who have investigated the interaction between viscous flow and the orography one can cite the work of Mason and Sykes (1978) who have studied the interaction of topography and Ekman boundary layer pumping in a stratified atmosphere and developed a two-dimensional theory for the prediction of the average pressure drag per unit area. These authors investigated homogeneous and stratified flows. In the case of homogeneous flows the Froude number was of order one and several Rossby number regimes were studied, namely 0.3, 1.0, 3.0 and 10.0.

Analytical work on the description of flow of an adiabatic boundary layer on a uniformly rough surface over a two-dimensional hump with a small curvature have been carried out by Jackson and Hunt (1975). Their efforts were concentrated on situations when high winds occur and stratification has a small effect on the flow near the surface of the hill. The horizontal scale of the hill considered varied from 100 meters to 10 kilometers and the aspect ratio was limited to be less than 0.05. Britter et al. (1981) have made wind tunnel experiments of turbulent flow over a rough two-dimensional hill placed in a neutrally stable boundary layer with a depth ten times the height of the obstacle and found that the analytical linearised results of Jackson and Hunt (1975) were in close agreement with their measurements of the streamwise mean and turbulent velocities for locations at or upwind of the hill but not in the wake. Mason and King (1985) have investigated the flow over a roughly circular hill whose height is 100 meters and its diameter is about 800 meters and have used their results to compare the linear analytic theory of Jackson and Hunt (1975).

Mason and King found that comparison of the linear theory of Jackson and Hunt for topographic slopes of about 24 degrees the theory gives good estimates of the flow speed increases but that the model fails to predict the extent of the velocity reduction in the lee of the orography.

In general, the studies mentioned above are no doubt important but they require simplifications in order to find analytical solutions, such as the assumptions of two-dimensional flow, and are concentrated on flow past small hills and not mesoscale mountains which are features having horizontal extensions varying from tens of kilometers to hundreds of kilometers and heights of the order of a thousand meters. Also the flow regimes investigated by most authors are high Froude number regimes, the value of the Froude number referred to here being the one in the free atmosphere. Thorpe et al. (1993) have investigated the interaction between the boundary layer and the orography for the particular case of flow past the Alps and have concluded that low-level potential vorticity anomalies are created as a result of such an interaction.

Masson and Bougeault (1996), reporting three dimensional numerical simulations of the IOP 10 of the Pyrenees Experiment concerning the cierzo which occurs in the Ebro valley, found that this wind follows qualitatively the Ekman theory of boundary layers. The wind is found to veer with height, although it is located over complex orography. Contrary to this result experimental studies performed by Noilhan et al. (1982), conducted on the Lannemezan Plateau located to the north of the Pyrenees show that when the direction of the upstream flow is such that the flow does not interact with the orography, the flow veers with height as predicted by Ekman, but when the flow has interacted with the orography the wind at the upper levels shows a tendency to deviate to the left.

Numerical investigations, through the use of a shallow-water model, on the effect of bottom friction in a low Froude number flow that interacts with an isolated obstacle have been performed by Grubisic et al. (1995), who observed that bottom friction acts as an inhibitor of vortex shedding. The results obtained with the Reading Model show that in the case of stratified flows the result of Grubisic et al. is incomplete.

2. The boundary layer

The concept of a boundary layer in fluid flows has been attributed to Froude, who carried out a series of laboratory towing experiments in the early 1870s to study the frictional resistance of a thin flat plate when towed in still water (Garrat, 1992). The term itself was perhaps first introduced into the literature by Prandtl, around 1905, working in the field of aerodynamics, who was concerned with the flow of a fluid of low viscosity close to a solid boundary. His research recognized the transition, through a thin aerodynamic boundary layer, from irrotational flow well away from the boundary to the condition of no-slip at the boundary (Garrat, 1992).

In the atmospheric context, it has always been difficult to define precisely what the boundary layer is. Nevertheless, a useful definition identifies the boundary layer as the layer of air directly on top of the Earth's surface in which the effects of the surface (friction, heating and cooling) are felt directly on time scales less than a day, and in which significant fluxes of momentum, heat or matter are carried by turbulent motions on a scale of the order of the depth of the boundary layer or less (Stull, 1988).

2.1. K-Theory

Some means are needed to determine the vertical dependence of the turbulent momentum flux divergence in terms of mean variables in order to obtain a closed set of equations for the boundary layer variables. It is usually assumed that turbulent eddies act in a manner analogous to molecular diffusion so that the flux of a given field is proportional to the local gradient of the mean. Thus, the turbulent flux terms are written as,

$$\langle u'v'\rangle = -K_m\left(\frac{\partial u}{\partial z}\right), \qquad \langle v'w'\rangle = -K_m\left(\frac{\partial v}{\partial z}\right)$$

and the potential temperature flux can be written as

$$\langle \Theta'w'\rangle = -K_h\left(\frac{\partial \Theta}{\partial z}\right)$$

Where K_m $(m^2 s^{-1})$ is the eddy viscosity coefficient and K_h $(m^2 s^{-1})$ is the eddy diffusivity of heat (Holton, 1992).

2.2 The Ekman boundary layer

Ekman's approximation to the Navier-Stokes equations was suggested to him by Fjortoft Nansen, who had returned from a trip across the Arctic (Brown, 1991). Nansen gave Ekman the information that the drift of the ice was always at a significant angle to the Earth's rotation. This observation was used by Ekman to simplify the Navier-Stokes equations to the point that he was able to find an analytic solution. Ekman was working with the oceanic planetary boundary layer, but he realized that the equations were also applicable to the atmospheric planetary boundary layer.

Ekman assumed that the flow in the planetary boundary layer was slowly varying (steady-state), horizontally homogeneous (eliminating the inertia terms) and there was negligible vertical motion relative to the horizontal and that the eddy viscosity coefficient was constant. He was left with a balance between the Coriolis, pressure gradient, and viscous terms. When the flow is near the surface, there is a source of mechanically generated turbulence, hence eddy viscosity, at the surface. A large magnitude of the eddy-viscosity coefficient K represents the large transport capability of the eddy motion. The no slip boundary condition, $u \to 0$, is a good approximation for the surface. Consequently near the surface the velocity shear is confined to a thin layer of depth H. Hence, the vertical stress term, $K\,\partial u/\partial z$, gets large near the surface. In this case, the Reynolds number uH/K can get arbitrarily small as the boundary is approached and H decreases. The result is that the viscous terms become important in the equations. In fact, as one gets very close to the surface (within a centimeter for the atmosphere), there is no room for turbulence and a laminar sublayer exists where only molecular viscosity is significant (Brown, 1991).

The question of how thick the geophysical turbulent boundary layer is would seem to involve the distribution of turbulence, hence the value of the eddy viscosity. The inviscid equations may be assumed to apply in the free atmosphere. It is assumed that, when K is taken to be a function of height, it decreases towards zero far away from where the turbulence is being created at the boundary. Ekman was able to show that the departure of
the planetary boundary layer velocity from the freestream velocity decayed exponentially with distance from the surface. Hence the planetary boundary layer is thin with respect to the depth of the troposphere (Brown, 1991).

Ekman's solution to the momentum equations applies for horizontally homogeneous, steady-state, two-dimensional flow in a rotating frame of reference. The main viscous forces are those associated with the vertical shear, $\rho K \partial u / \partial z$. Since the vertical shear becomes small in the freestream, the value of K becomes unimportant at the top of the planetary boundary layer. For this reason, a constant K assumption yields an adequate first approximation for the planetary boundary layer equations. The only requirement is that there be an eddy continuum. When the boundary layer effects are ignored, the solutions of the momentum equations ultimately fail in some respect. Frequently the success or failure of a large-scale numerical integration of the complete Navier-Stokes equations will depend on how well the boundary layers are parameterized, Brown (1991).

2.2.1 Ekman layer instability

Brown (1972), states that an inviscid or slightly viscous fluid cannot support a local extremum in vorticity and that similarly, when a geostrophic flow is uniform on a mesoscale such that the Coriolis force acts to turn the flow (to a 45 degree angle in the Ekman layer, since the flow veers with height from the surface where the flow is at rest due to the no-slip boundary condition up to the top of the boundary layer where the flow reaches the value of the background speed, $4ms^{-1}$), the internal stress field (viscosity) cannot support the turning above a minimum velocity. This turning of the mean flow produces two-dimensional velocity profiles with inflection points that are indicative of a vorticity extreme in that planar cross-section. Thus the flow becomes unstable and rolls over to realign with the mean flow. Hence the inflection point instability, at least in two-dimensional calculations seems to be the cause of roll vortices.

Usually, there are two primary sources of energy, or mechanisms, by which an instability can draw energy from the mean flow in the atmosphere. One is gravitational instability, related directly to the minimum potential energy criteria for stability. Examples are baroclinic and convective instabilities such as Rayleigh-Bénard convection. The other source of energy lies in the velocity shear instabilities such as barotropic instability, or Kelvin-Helmholtz waves. The latter are frequently indicated in the atmosphere by 'coat-hanger' cloud formations on top of a shear layer.

Etling and Brown (1993), have suggested that perhaps, strictly two-dimensional theory is not applicable to the inflection point problem in a rotating boundary layer and that inflection points are very weak in a turbulent boundary layer. The simulations performed with the Reading Model using viscous flow without diffusion of the temperature field over flat terrain have not shown the production of roll vortices, the inflection point instability is either not present at all or is very weak and can be neglected.

3. Numerical simulations

The initial velocity profile used corresponds to the solution to the Ekman equations for an eddy-viscosity coefficient equal to $5m^2s^{-1}$. This value was chosen because it yields a boundary layer whose depth is a fraction of the orography. This will produce a flow whose local Froude number changes with height and whose value outside the boundary layer is equal to 0.2. It has been shown, Smolarkiewicz and Rotunno (1989), for example, that vortices are shed for this low Froude number regime and, perhaps, this will allow vortices to be shed outside the boundary layer. It is sought to investigate, amongst other things, if vortices are shed when a boundary layer is present in the flow.

The profile is of the form:

$$u = u_g\left(1 - e^{-\gamma z}\cos(\gamma z)\right) \tag{1}$$

$$v = u_g e^{-\gamma z} sin(\gamma z) \tag{2}$$

where $u_g = 4ms^{-1}$ is the background speed and $\gamma = (f/2K)^{1/2}$. The depth of the Ekman boundary layer is given by $D_e = \pi/\gamma$ and is close to one kilometer. For simplicity the temperature field is not subjected to diffusion. Only the windfield has been diffused.

A numerical experiment whose initial velocity and temperature profiles were the same as those mentioned here was performed over flat terrain in order to check the stability of the flow. No oscillations were observed in the flow. This allows one to infer that any kind of oscillation found when integrating the momentum equations in the presence of orography will be entirely due to the interaction between the flow and the orography.

3.1. The model momentum equations

The momentum equations solved in the numerical model are, as discussed in Gutiérrez (1997),

$$\frac{Du}{Dt} = -\frac{\partial \phi}{\partial x} + \frac{\partial \varphi'}{\partial \sigma}\frac{\sigma}{p_*}\frac{\partial p_*}{\partial x} + fv + D_u \tag{3}$$

$$\frac{Dv}{Dt} = -\frac{\partial \phi'}{\partial y} + \frac{\partial \varphi'}{\partial \sigma}\frac{\sigma}{p_*}\frac{\partial p_*}{\partial y} - fu + D_v \tag{4}$$

$$\frac{D\tilde{w}}{Dt} = -S\frac{\partial \varphi'}{\partial \sigma} - g\frac{\theta'}{\theta_s} + D_w \tag{5}$$

The diffusion terms, expressed in height coordinates are of the form:

$$D_i = K\frac{\partial^2 v_i}{\partial z^2} \tag{6}$$

where K is the constant eddy-viscosity coefficient and v_i represents any of the three components of the velocity field $V = (u, v, w)$.

An Ekman type of balance is not expected in the area close to the orography since the flow is expected to be influenced by the presence of the orography. The Ekman approximation to the momentum equations is valid only when far away from the orography since Ekman assumed a level surface for his simplification of the Navier-Stokes equations.

Given that not much is known about the structure and dynamics of the atmospheric boundary layer over irregular terrain it is best to use a simple boundary layer parametrisation in order to get an idea of the main features of the dynamics/interaction between the orography and the flow. The importance of this investigation lies in the fact that viscous mechanisms are dominant over inviscid mechanisms in the lower troposphere. Most of the research dealing with flow past isolated mountains found in the literature is concerned with inviscid simulations, it may well be that inviscid mechanisms of vorticity production are less important than viscous sources of vorticity.

In the numerical experiments presented here a no-slip lower boundary condition has been implemented, and an explicit parametrisation of the diffusion has been added to the momentum equations only, i.e., the fields u and v are diffused, whereas the temperature field is not in order to comply with the Ekman approximation when far away from the orography. The simulation takes place on an f-plane located at 40 degrees north of the equator.

The orography used is a bell-shaped mountain with a circular base of radius 50 kilometers and a height of 2 kilometers. The eddy-viscosity coefficient chosen is equal to $K = 5m^2s^{-1}$, which yields an Ekman layer depth of about 1 kilometer. The background speed of the air in the free atmosphere is equal to $u = 4ms^{-1}$, the stratification of the atmosphere is $N = 10^{-2}s^{-1}$. The background Froude number in the free atmosphere, in this simulation, is consequently equal to 0.2 and the Rossby number R_o is equal to 0.4.

4. Vortex shedding

A calculation of 72 hours shows that no vortex shedding has taken place at 400 metres above level ground. The result of this calculation is shown in Figure 1 and corresponds to flow whose Rossby number is 0.4. Hence, even though there is background rotation and the upstream flow is not symmetric, conditions shown to lead to vortex shedding by Sun and Chern (1994) the lee vortices remain steady, at least at this height. This shows that the presence of a boundary layer can potentially inhibit the shedding of vortices.

This agrees with the results of Grubisic et al. (1995), who performed simulations of flow past an isolated bell-shaped mountain with a circular base with a shallow water model in which they had included the effects of surface friction. They did not include background rotation in their calculations, except in the case of a simulation using the orography of Hawaii. They explained in this way why the lee vortices observed in Hawaii, show no tendency to shed. These investigators found that the bottom friction had a strong impact on the mean flow characteristics, reducing the width and strength of the velocity deficit region in the wake as well as the wake length. They also found that increasing friction stabilises the wake, first near the obstacle by preventing classical eddy shedding, and later by eliminating the remaining unsteadiness from the leeward end of the wake.

Grubisic et al. (1995), explain the weakening of the vorticity field in their simulations as been due to the destruction of vorticity by bottom friction and because the imposed downstream pressure gradient in their simulation strongly opposes the reversed flow in the wake region. An interesting point in their results is that even though they found that bottom friction remained dominated by a pseudoinviscid process related to the presence of hydraulic jumps, no evidence of such hydraulic jumps has been found in the calculations presented in this paper.

Note however that Grubisic et al. (1995), used a shallow water model and hence did not simulate a continuously stratified viscous flow interacting with an isolated obstacle. In the continuously stratified case the local Froude number varies with height, as shown in Figure 2 and it becomes possible for the lee vortices to be shed. Figure 3 shows the vertical vorticity field for the same simulation after 72 hours of integration at a height of 1000 meters above level ground. The vortices appear to be shedding at this height.

The vertical vorticity field at 1500 metres above level ground shows a better defined shedding event. This is shown in Figure 4. These results show that the presence of a boundary layer may inhibit the shedding of vortices in the lower layers of the flow but that the shedding may take place at higher levels.

No signal associated with shedding was detected in the meridional component of the surface pressure drag. Indicating that the fact that the lee vortices remain attached to the orography at the lower levels of the flow has a dominant influence in the surface pressure drag.

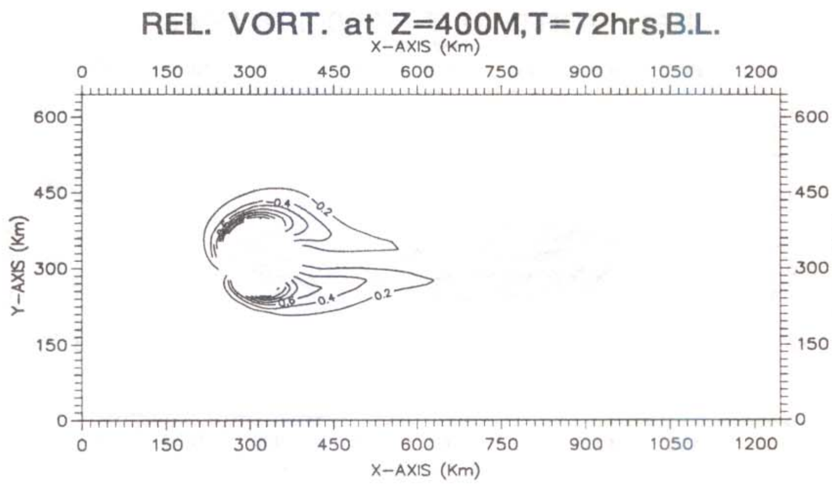

Fig. 1: Vertical vorticity field at a height of 400 meters above level ground. Contour interval $0.2 \times 10^{-4} s^{-1}$. Integration time 72 hours. Rossby number 0.4.

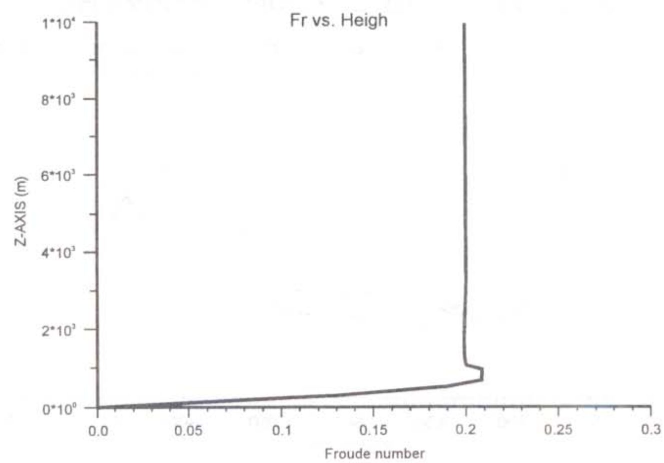

Fig. 2 : Variation with height of the local Froude number obtained from an Ekman velocity profile.

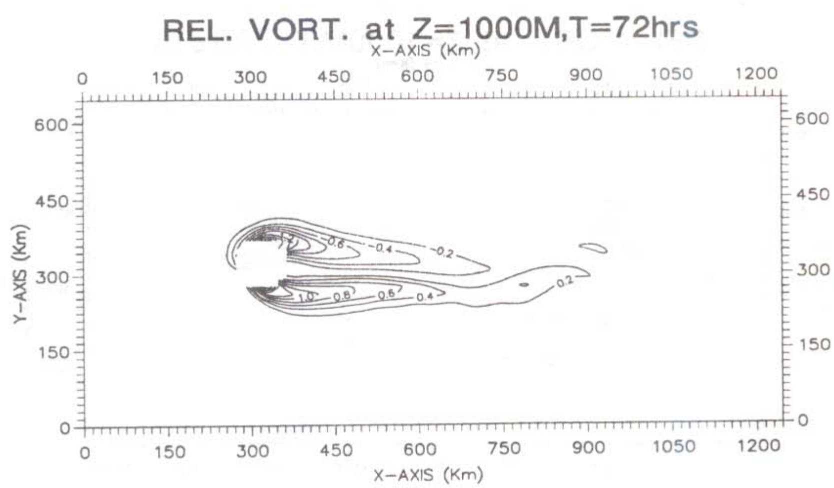

Fig. 3: Vertical vorticity field at a height of 1000 meters above level ground. Contour interval $0.2 \times 10^{-4} s^{-1}$. Integration time 72 hours. Rossby number 0.4.

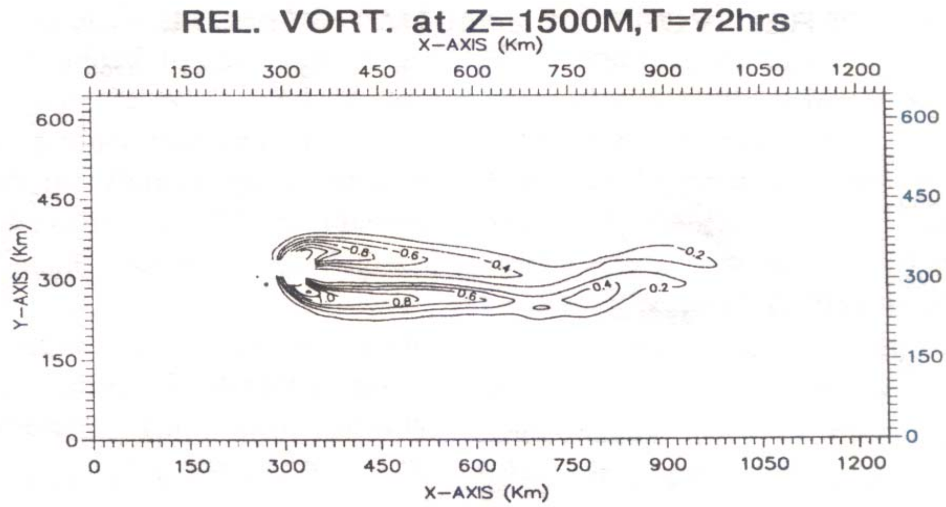

Fig. 4: Vertical vorticity field at a height of 1500 meters above level ground. Contour interval $0.2 \times 10^{-4} s^{-1}$. Integration time 72 hours. Rossby number 0.4.

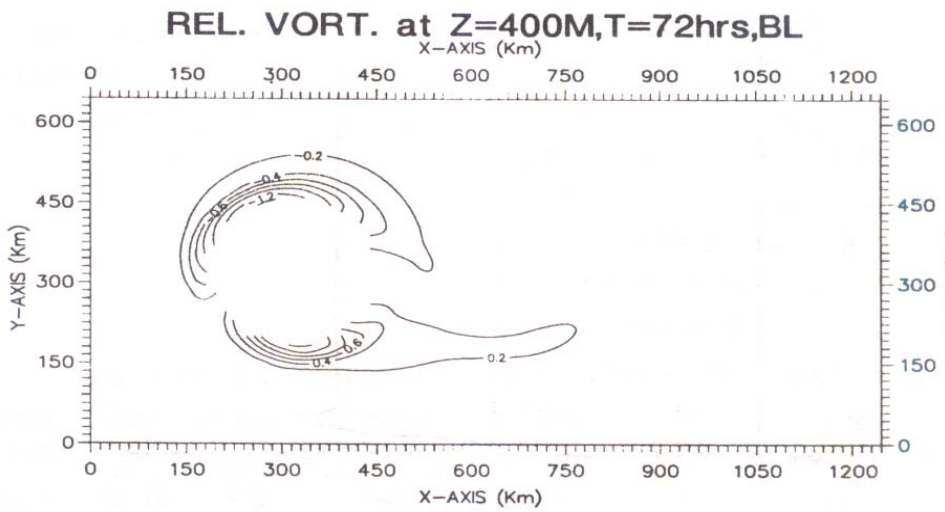

Fig. 5: Relative vertical vorticity at a height of 400 meters above level ground. Contour interval $0.2 \times 10^{-4} s^{-1}$. Integration time 72 hours. Rossby number 0.2.

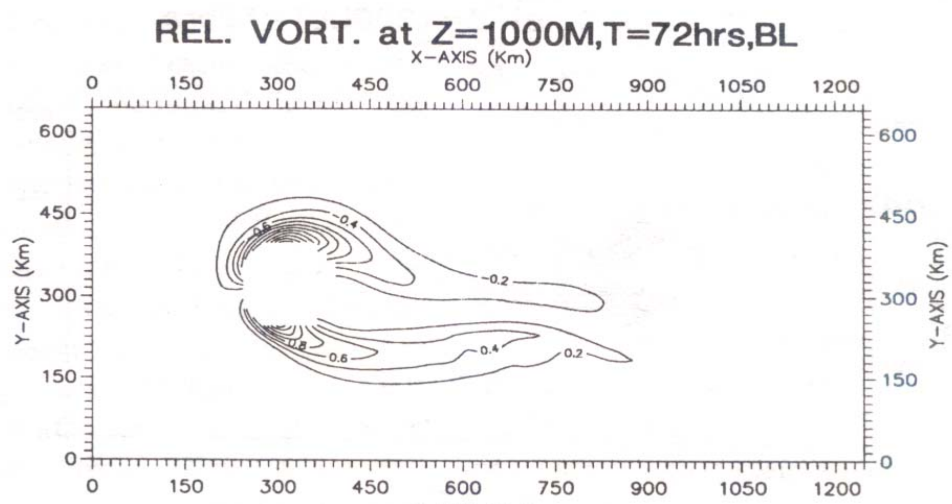

Fig. 6: Relative vertical vorticity at a height of 1000 meters above level ground. Contour interval $0.2 \times 10^{-4} s^{-1}$. Integration time 72 hours. Rossby number 0.2.

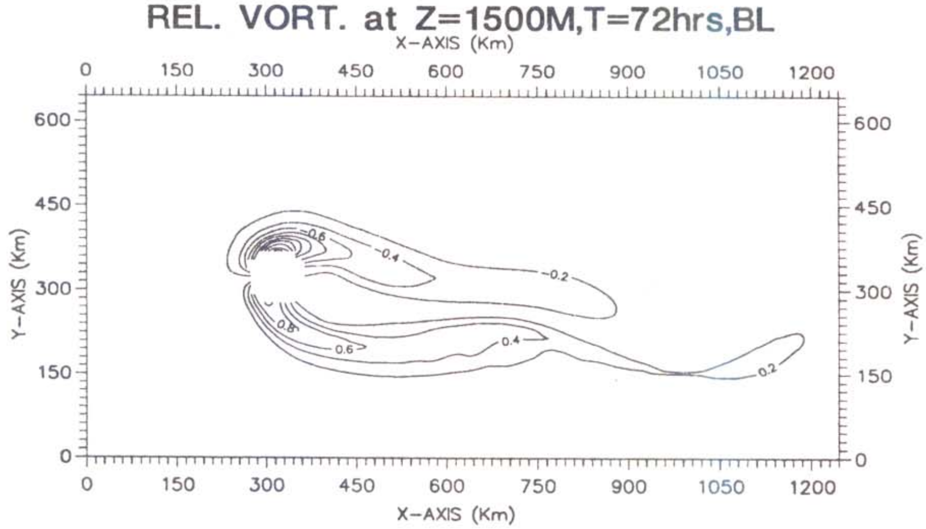

Fig. 7: Relative vertical vorticity at a height of 1500 meters above level ground. Contour interval $0.2 \times 10^{-4} s^{-1}$. Integration time 72 hours. Rossby number 0.2.

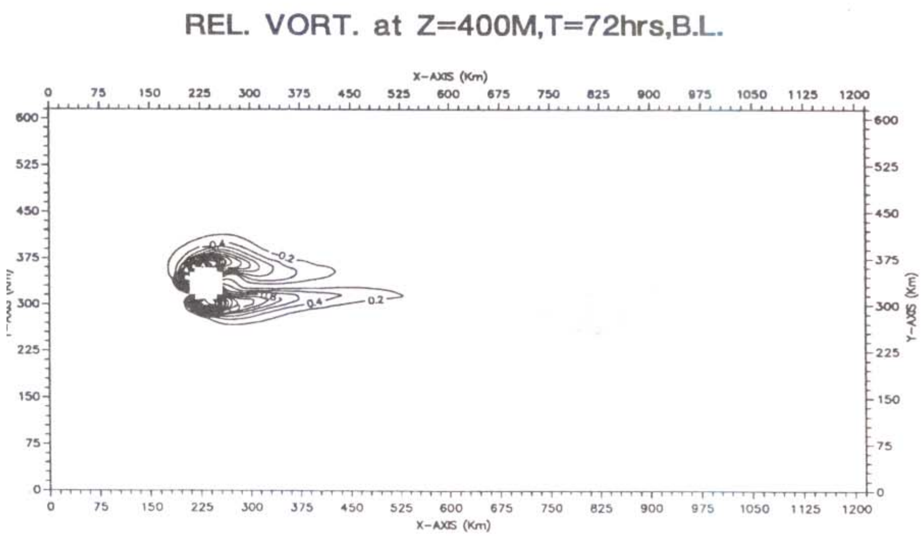

Fig. 8: Relative vertical vorticity at a height of 400 meters above level ground. Contour interval $0.2 \times 10^{-4} s^{-1}$. Integration time 12 hours. Rossby number 0.8.

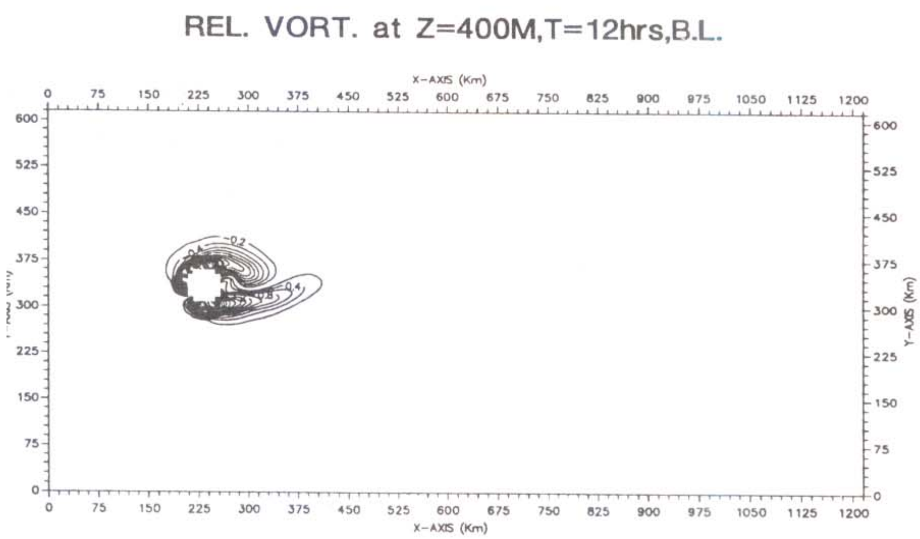

Fig. 9: Relative vertical vorticity at a height of 1000 meters above level ground. Contour interval $0.2 \times 10^{-4} s^{-1}$. Integration time 72 hours. Rossby number 0.8.

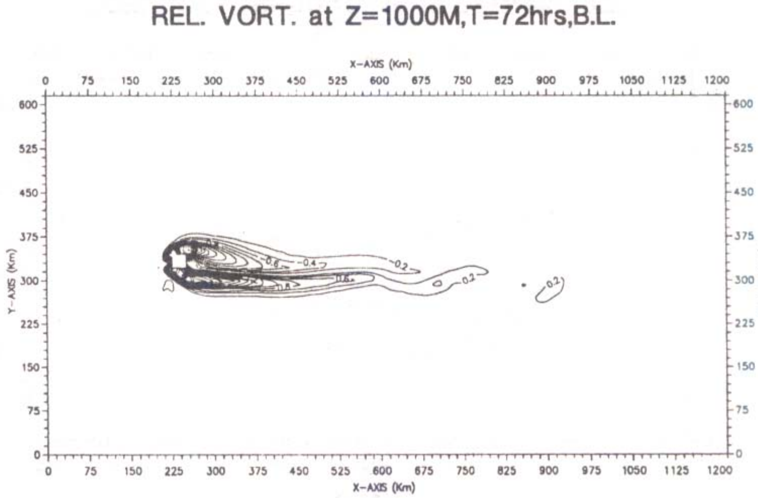

Fig. 10: Relative vertical vorticity at a height of 1000 meters above level ground. Contour interval $0.2 \times 10^{-4} s^{-1}$. Integration time 72 hours. Rossby number 0.8

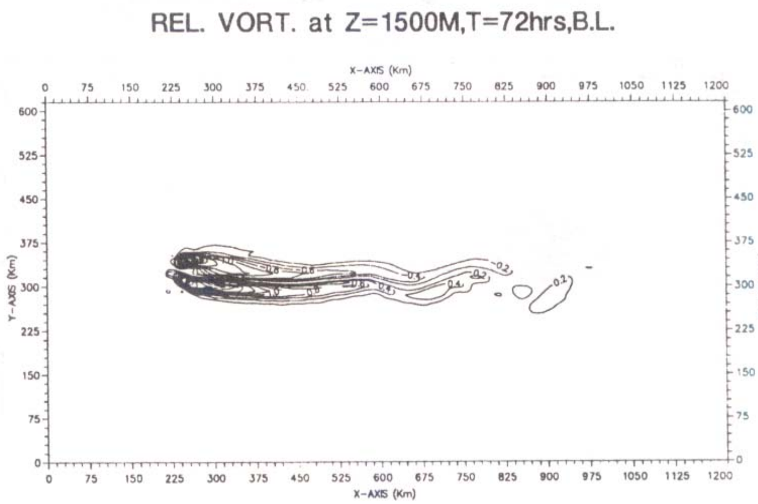

Fig. 11: Relative vertical vorticity at a height of 1500 meters above level ground. Contour interval $0.2 \times 10^{-4} s^{-1}$. Integration time 72 hours. Rossby number 0.8.

5. Effects of different Rossby numbers

The effects of a different aspect ratio on the flow will be investigated in this section. Since this simulation has background rotation then a different aspect ratio can be associated with a different Rossby number regime.

In the first case to be investigated, results obtained with an obstacle whose diameter is twice as big as the one used at the beginning of this paper will be presented and discussed. For this case the Rossby number associated with the flow is equal to 0.2.

A twelve-hour integration, not shown, yields two centers of vorticity located, as in the viscous flow simulation with $R_o = 0.4$, at the flanks of the orography. The area of anticyclonic vorticity has, for the $R_o = 0.2$ experiment, extended deeper into the southern part of the lee. This may be due to the effect of the background rotation in the flow since the advection time has doubled in this experiment due to the doubling in size of the orography's radius.

An integration of 72 hours shows that at a height of 400 meters above level ground no vortex shedding has taken place. The vorticity field is shown in Figure 5. this is similar to the result

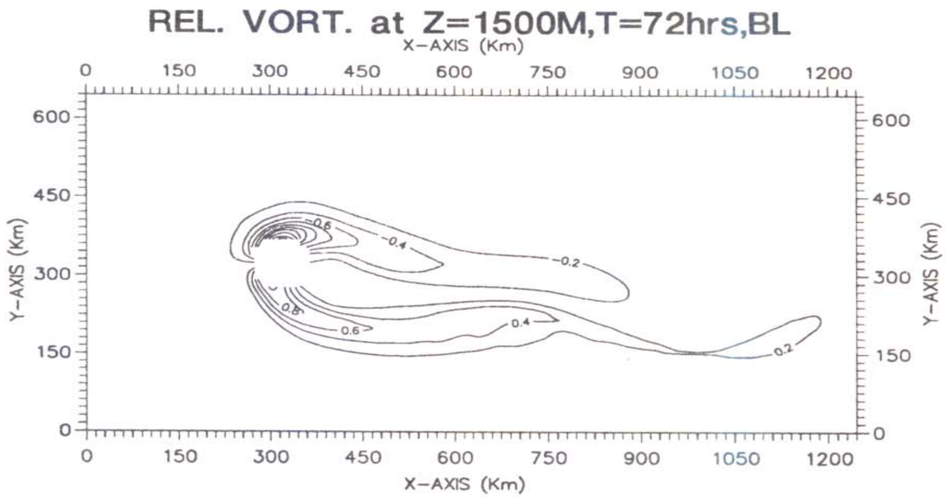

Fig. 7: Relative vertical vorticity at a height of 1500 meters above level ground. Contour interval $0.2 \times 10^{-4} s^{-1}$. Integration time 72 hours. Rossby number 0.2.

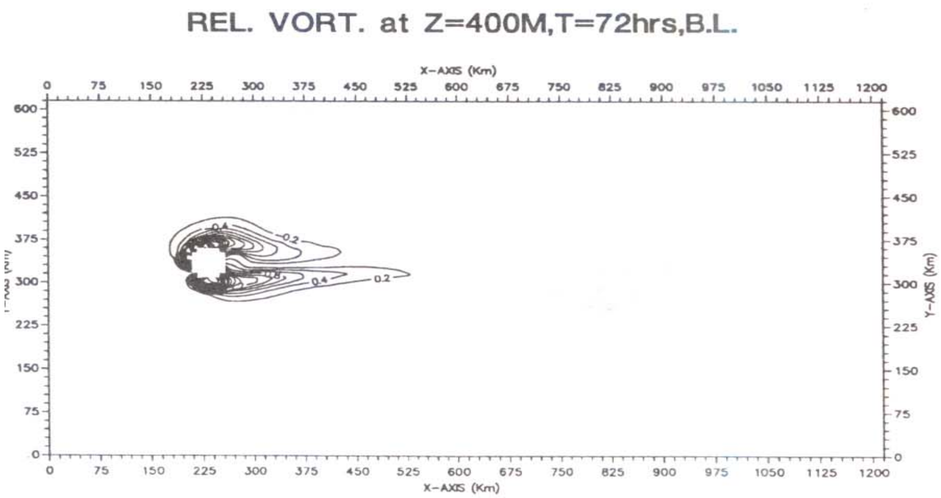

Fig. 8: Relative vertical vorticity at a height of 400 meters above level ground. Contour interval $0.2 \times 10^{-4} s^{-1}$. Integration time 12 hours. Rossby number 0.8.

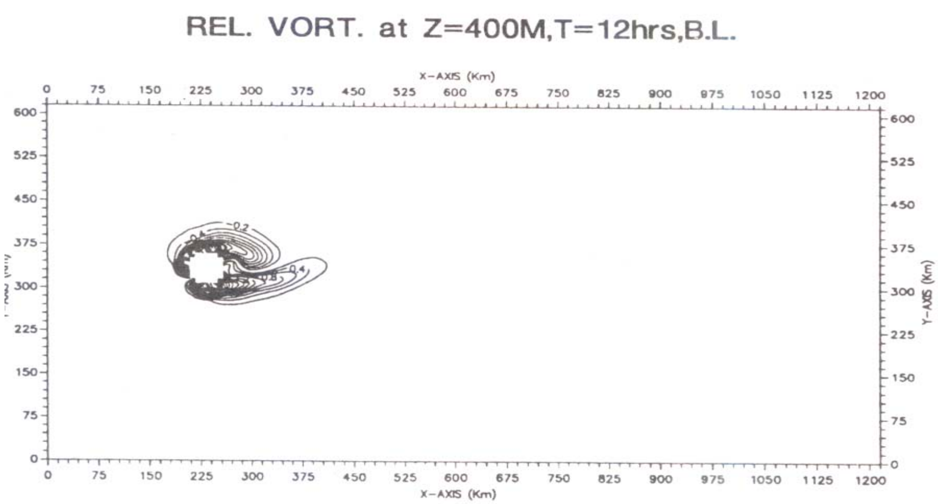

Fig. 9: Relative vertical vorticity at a height of 1000 meters above level ground. Contour interval $0.2 \times 10^{-4} s^{-1}$. Integration time 72 hours. Rossby number 0.8.

found in the previous simulation where the Rossby number was equal to 0.4.

The vertical vorticity field at 1000 metres above level ground shown in Figure 6 indicates that the lee vortices are more elongated than those at 400 metres. But still no clear evidence of vortex shedding is found at this level.

At a height of 1500 metres above level ground the southern vortex begins to separate from the orography and is advected downstrem. This is shown in Figure 7. Vortex shedding is then seen to occur first at higher levels in the flow in the experiment with a flow whose Rossby number is 0.2 than in the experiment with a flow whose Rossby number is 0.4.

The third obstacle investigated has a radius of 25 kilometres, consequently the Rossby number for this flow is equal to 0.8. The horizontal resolution was increased in order to be able to resolve this smaller obstacle. The separation between consecutive gridpoints in the horizontal was chosen to be $dx = dy = 8km$ and the temporal resolution was chosen to be equal to 8 seconds. The vertical vorticity field obtained after a simulation of twelve hours is shown in Figure 8. One can observe there that the horizontal dimensions of the vortices are smaller than those observed for the other two experiments with wider obstacles obtained at the same integration time.

An integration of 72 hours shows that at 400 metres above level ground, shown in Figure 9 the vortices have not been shed, but they show an elongation in the zonal direction. At a height of 1000 metres above level ground the vortices have been shed, this is shown in Figure 10. Notice that at this level in the simulations with 'flatter' obstacles no such separation of the vortices had taken place. This indicates that steeper obstacles appear to favour the shedding of vortices.

At 1500 metres above level ground, the shed anticyclonic/cyclonic vortex centres show smaller/greater values than those at 1000 metres above level ground. This is shown in Figure 11.

6. Concluding remarks

Contrary to the results of Grubisic et al. (1995), vortex shedding does occur when viscous flow (boundary layer flow) interacts with orography. This result is closer to what actually happens in the lower troposphere where lee vortices are shed in the presence of a boundary layer.

The larger the Rossby number of the flow the larger the frequency of vortex shedding in the flow investigated. This occurs at levels located at one half and at three fourths of the mountain's height. This is due to the increase with height of the local Froude number of the flow due to the weakening with height of the boundary layer turbulence.

The meridional extension of the lee vortices is found to be proportional to the cross section of the orography.

RESUMEN

Se analizan las interacciones entre flujo viscoso y la orografía y se investigan las circunstancias bajo las cuales se crean calles de vórtices para un conjunto de montañas con dimensiones horizontales distintas. Se encuentra que en los bajos niveles del modelo (un quinto de la altura de la orografía) los vórtices no son advectados corriente abajo mientras que en niveles situados a dos y a tres cuartos de la altura de la orografía se observa la formación de calles de vórtices. Estas simulaciones han sido hechas por medio de un modelo numérico tridimensional, no lineal y no hidrostático. El flujo investigado es continuamente estratificado.

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