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Stratified flow past a mesoscale mountain range

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Resumen

Se investiga el efecto de la interacción entre el viento y una cordillera aislada orientada de forma perpendicular al flujo de referencia. La simulación numérica tridimensional muestra que para el régimen de viento escogido se generan ondas de montaña. Se comparan los resultados numéricos obtenidos con observaciones de ondas de montaña hechas sobre el territorio costarricense.

1. Introduction:

Mountain waves were not really known before 1935 when glider pilots first used them in their flights. Küttner in 1939 noted the close analogy between them and the standing barrier waves observed on the free surface of a river flowing over a corrugation at the bed.

Several authors, Smith (1979), Smolarkiewicz and Rotunno (1989), Queney (1948), Hoinka (1985), Athanassiadou (1995), Miranda (1990), have stressed the important influence of mountains over the atmospheric flow at both the synoptic scale and the mesoscale (give a list). The case of an isolated mountain and an infinite mountain range have been the subject of intense research. However, the three-dimensional problem of flow past a mesoscale obstacle has been dealt with less frequently.

Mountain waves have been documented in several areas of the planet. In this article, theory, simulation and observation of mountain waves will be discussed.

The results of the numerical simulations presented in this paper were performed by means of a Sun workstation at the Laboratory for Atmospheric and Planetary Research of the University of Costa Rica.

Research on mountain flows is important in a mountainous country like Costa Rica in order to be able to determine the conditions under which strong winds and turbulence can develop. The enhancement of rain is also of importance, in particular for the prediction of flash floods and landslides.

2. Two-dimensional versus three-dimensional flows

Several authors have tried to compare linear theory results with observations. As far as lee-waves are concerned the use of Scorer's criterion has been successful (Smith 1979), statistically speaking, for the prediction of the generation of lee waves.

Consider the flow over a three-dimensional obstacle that is very long and approximately homogeneous in the

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direction perpendicular to the upcoming flow. If such an obstacle has width 2Y and downstream length 2X where X<<Y, and the flow consists of a stratified fluid whose Brunt-Väisälä frequency is represented by the letter N and has a mean velocity U.

Such a flow generates internal waves with dominant vertical wavenumber scale of $n \approx N/U$ (Baines, 1995). If the horizontal length-scale happens to be longer than the vertical so that $k^2 + m^2 << N^2/U^2$ then the horizontal component of the group velocity is of magnitude $N/n \approx U$.

Waves with $n < N/U$ have larger group velocity and are able to propagate against and across the stream. In the central region of the obstacle, the individual evolution of the flow is the same as for two-dimensional obstacles, but in zones near the ends of the obstacle the flow is three-dimensional. Now, due to the fact that the characteristic group velocity of the significant waves is equal to U, the influence of the northern and southern ends of the topography reaches the centre in a time of order y/U. Till this time the flow in the central region is nearly two-dimensional and afterwards it evolves towards its three-dimensional form. In the case treated here $Y = 200km$ and $U = 20m/s$ so the time is equal to 2.8 hours.

It is important to realise that in order to be able to use numerical models in forecasting regional weather in Central America, knowledge of mountain flow dynamics and high horizontal resolution of the terrain are important. The high resolution is needed in order to correctly model the forcing due to the mountains.

The interaction of flow with orography is a very active area of research, and currently there are important efforts in trying to accurately model the forcing of orography in global models, Lafore et al. 2000, Andrae, 2000.

3. The computer model used

The model used to perform the simulations was developed at the University of Reading by Dr. Pedro Miranda (1990). A description of this model, that will be called the Reading model from here onwards is given by several authors, Miranda (1990), Miranda and James (1992), Athanassiadou (1995) and Gutiérrez (1997).

The model is three-dimensional, dry, non-linear, non-hydrostatic and is started impulsively. It has terrain following coordinates (σ - coordinates) and approximates the derivatives in the equations by means of centred differences.

The Reading model has radiation boundary conditions at the lateral boundaries and a sponge layer at the top of the domain of integration. It also uses time and spatial filters in order to eliminate the computational mode and the instabilities connected with subgrid scale waves.

4. Theory of mountain waves

Any ground corrugation produces a deformation in the airflow aloft. In general, the properties of this orographic perturbation largely depend on its horizontal scale, itself mainly determined by the horizontal extent of the corrugation in the direction of the wind.

The simplest model of mountain waves consists in a two-dimensional flow interacting with an infinitely long mountain range oriented such that it is perpendicular to the background flow. Linearising the momentum equations the following couple of equations are obtained,

$$\overline{u}\frac{\partial u'}{\partial x} + w'\frac{\partial \overline{u}}{\partial z} + \frac{\partial \varphi'}{\partial x} = 0 \qquad (1)$$

$$\overline{u}\frac{\partial w'}{\partial x} + \frac{\partial \varphi'}{\partial z} = b' \qquad (2)$$

where $\varphi' = p/\rho_o$ is a reduced pressure and $b' = g\theta'/\theta_o$ is the buoyancy.

The thermodynamic equation becomes,

$$\overline{u}\frac{\partial b'}{\partial x} + w'N^2 = 0 \qquad (3)$$

and the continuity equation becomes,

$$\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0 \qquad (4)$$

In order to get a wave equation in w' the pressure gradient will be eliminated by taking $\frac{\partial(1)}{\partial z} - \frac{\partial(2)}{\partial x}$, this yields,

$$\frac{\partial^2 w'}{\partial x^2} + \frac{\partial^2 w'}{\partial z^2} + w'\left[\frac{N^2}{\overline{u}^2} - \frac{1}{\overline{u}}\frac{\partial^2 \overline{u}}{\partial z^2}\right] = 0 \qquad (5)$$

in general the term in brackets is known as the Scorer parameter and denoted as l^2. This relation was first discovered by Scorer in 1949.

If it is assumed that the general solution to the wave equation is if the form,

$$w'(x,z) = w_1(z)\cos(kx) + w_2(z)\sin(kx) \quad (6)$$

and substituting (6) into (5) the equation becomes

$$\frac{\partial^2 w_i}{\partial z^2} + (l^2 - k^2)w_i = 0 \quad (7)$$

It can be seen in (7) that depending on the relative magnitudes of the Scorer parameter and the zonal wave number, which is a measure of the horizontal extent of the orography, vertically propagating mountain waves or vertically decaying mountain waves can be obtained. In case waves are not able to propagate upwards they will be advected downstream carrying with them large amounts of energy. These are known as lee-waves and can cause severe structural damage on buildings located on the lee side of the orography.

5. The simulation

The domain of integration consists of 121 gridpoints along the zonal direction and 91 gridpoints in the meridional direction. The separation between two successive grid points was chosen to be equal to 15 kilometres, this translates into an area of 1815 km by 1365 km. The number of vertical levels was chosen to be equal to thirty. The orography is a mountain range perpendicular to the zonal flow. The mean flow is purely zonal and inviscid. The mountain has a meridional extension of 200 km, a zonal width of 40 km and a height of 1 km. These dimensions were chosen in order to simulate flow about an orography that is similar in size to the northern section of the mountain range that crosses Costa Rica. The idealised orography used describes an elongated mountain aligned in the south-north direction with east-west and north-south symmetry. The mountain profile is given by the formula,

$$h(x,y) = \max\left(\frac{h_m + h_c}{\left(\left(1 + \frac{x^2 + (\max(|x| - x_m, 0))^2}{a^2}\right)\right)^2} - h_c, 0 \right) \quad (8)$$

where a is the horizontal scale over which the mountain elevation diminishes, h_m is the maximum height of the mountain (with a flat top), h_c is the cut-off height which determines the horizontal location at which the orography becomes zero.

This region is important amongst other things because in it is located the biggest dam in the country. The lago de Arenal is also the most important water reservoir of the country so evaporation in the lake forced by strong winds is also of importance. In the western part of the lake are located wind towers which produce electricity. Also the most active volcano of the country is situated in this area and it is important to have an idea about the trajectories of pollutants from the volcano could be deflected by the interaction of the upstream flow with the orography. The background flow has a speed of 20 m/s and a background Brunt-Väisälä frequency of 0.1 1/s This yields a flow whose background Froude number is equal to 2, this means that mountain waves are likely to form when the flow interacts with the orography. The simulation is carried out for a time interval of fifteen hours. After such a time waves can be seen to have been generated on top of the orography. This can be seen in Fig 1. Where a vertical cross section of the flow passing through the centre of the orography is shown. Layers where the perturbation potential temperature is negative are areas where air from lower layers is ascending. Likewise, areas where the perturbation potential temperature is positive are areas where air from upper layers is descending. The graphics were made by means of the Grid Analysis and Display System GrADS.

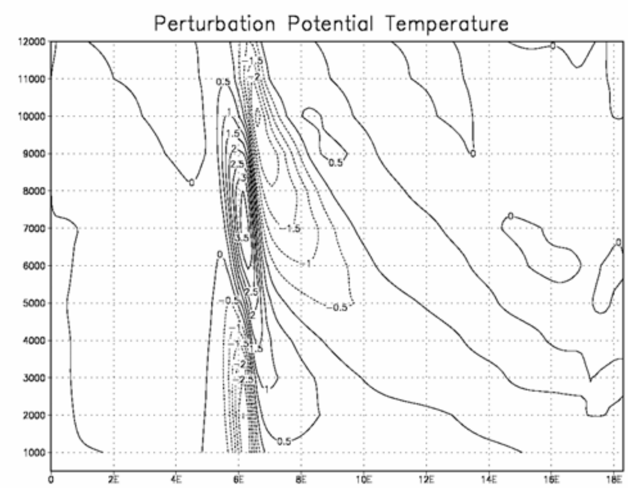

Figure 1: Perturbation potential temperature field in K in a vertical section passing though the centre of the orography. Integration time 15 hours.

The existence of mountain waves in this calculation can also be made evident by looking at other fields such as the streamlines shown in Fig 2. In this case the figure was prepared by exaggerating the vertical component of the velocity field by a factor of ten.

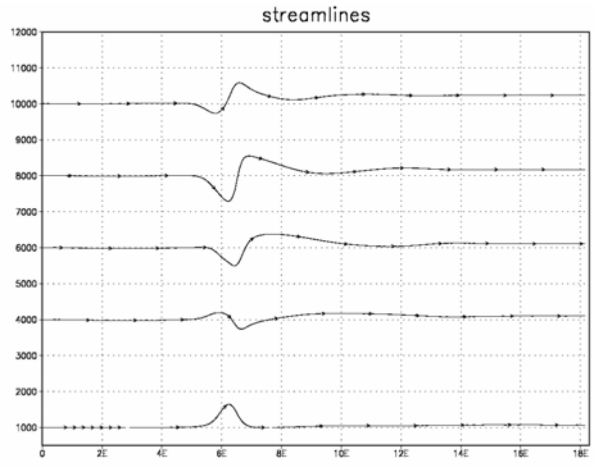

Figure 2: Streamlines in a vertical cross section passing through the centre of the orography. Integration time 15 hours.

Another field of interest is the zonal component of velocity. This one is shown in Fig. 3. It can be seen that the zonal component of velocity suffers accelerations and decelerations that are tantamount to the existence of turbulence created by the ascending mountain waves.

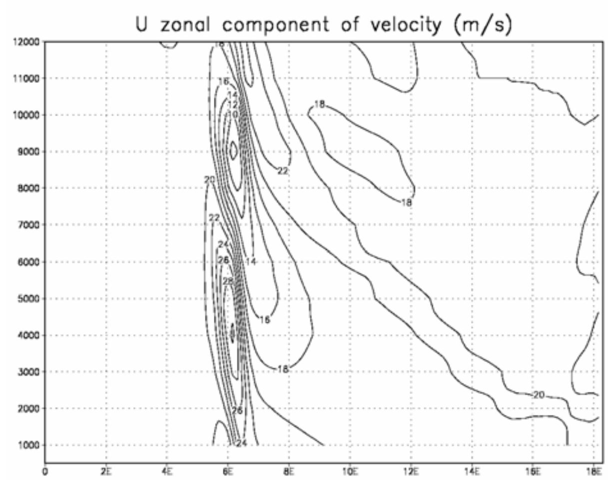

Figure 3: Cross-section of the zonal component of velocity on a plane passing through the centre of the orography.

The Scorer parameter is the equivalent of a refraction index for internal gravity waves. The vertical profile of the Scorer parameter l^2 is a good indicator of the gravity wave propagation in the corresponding atmospheric layers. Propagation can only occur if $l^2 > k^2$, where k is the horizontal wavenumber. Since the cross-section length of the orography is 40 kilometres the wavenumber is about $1.6x10^{-4} m^{-1}$, which means that its square value is of the order of 10^{-8}. Negative values in Fig. 4 are indicative of layers where waves have difficulty moving upwards whereas positive values of the Scorer parameters show layers where waves can propagate in the vertical without difficulty. The result shown in figure 4 was obtained after a 15 hour simulation.

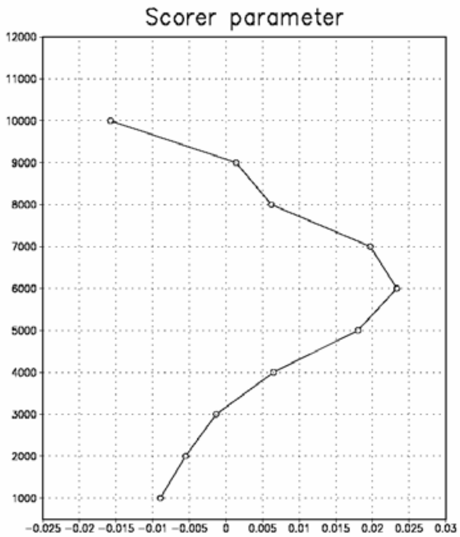

Figure 4: Variation of the Scorer parameter with respect to height, taken at a point located at the centre of the orography.

The perturbed zonal component of velocity field shown in figure 5 indicates that the effect of the orography is to reduce the speed of the flow on the upstream side of the mountain and, once mountain waves are produced the flow is accelerated on the downstream side of the mountain.

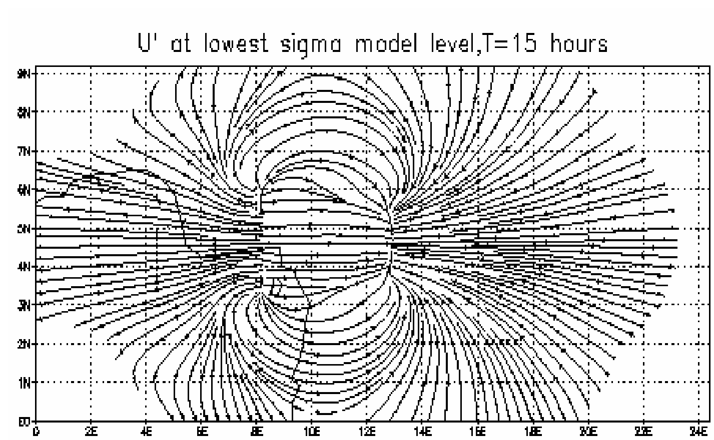

Figure 5: Zonal component of the velocity perturbation field shown on the first sigma level of the model.

Another source of information about the dynamics of the flow comes from looking at the structure of some fields on a cross section in the meridional direction.

The cross section along the meridional plane passing close to the centre of the orography shows that there is deceleration of the flow in the lower layers of the atmosphere and acceleration of the flow in the upper layers of the atmosphere. This is shown in figure 6.

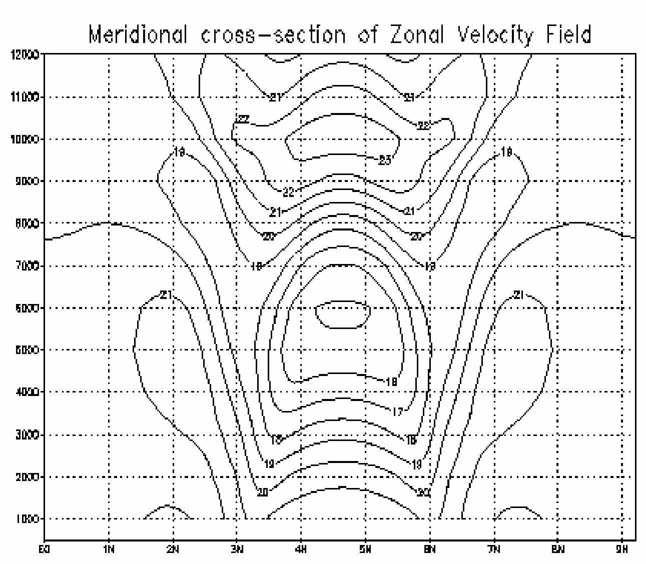

Figure 6: Meridional cross section of the zonal component of the velocity field on a plane close to the centre of the mountain.

The cross section along a meridional plane passing close to the centre of the orography of the meridional component of the velocity field shows a very interesting structure. This is shown in figure 7. Where it can be seen that at the lower layers of the atmosphere. The flow at the southern flank of the mountain develops a northern component as it passes around the orography while at the northern flank of the mountain the flow develops a southerly component generating thus an area of convergence in the lower layers of the flow.

At the upper layers of the flow the pattern shifts and as the flow moves past the mountain it develops a southerly component on the southern part of the domain and a northerly component on the northern part of the domain. This is most likely due to the low level convergence that, as a consequence of the conservation of mass generates divergence of the flow at upper levels.

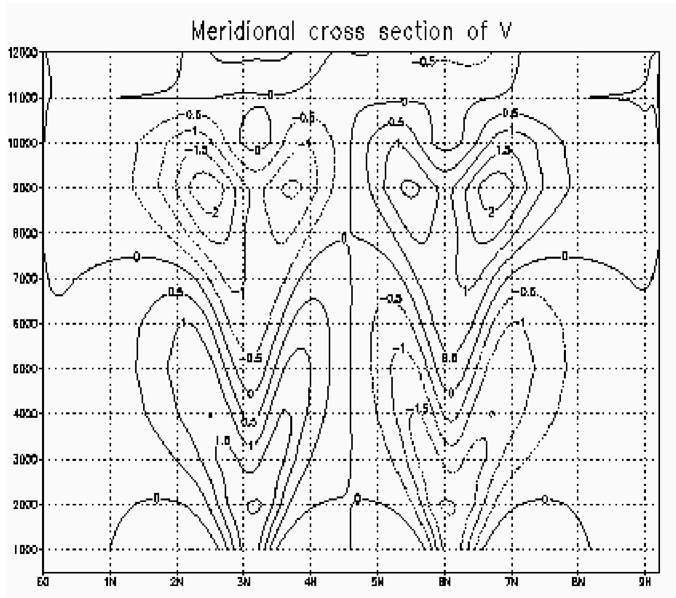

Figure 7: Meridional cross section of the meridional component of the velocity field passing near the centre of the mountain.

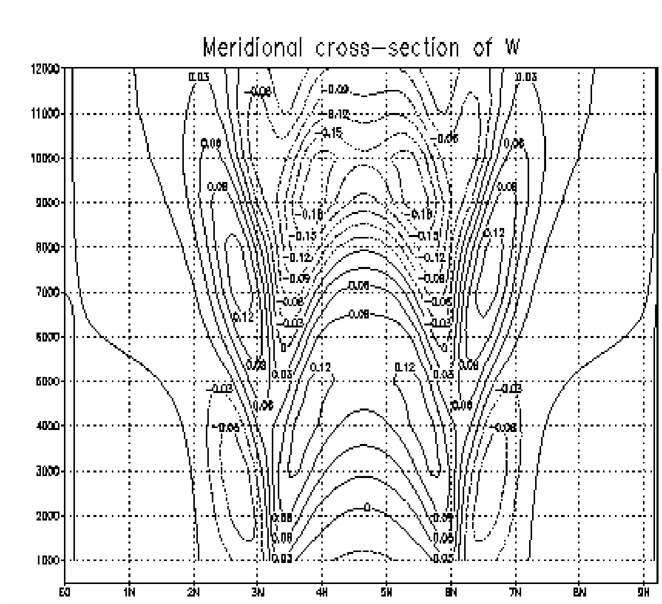

Figure 8: Meridional cross section of the vertical velocity field passing near the centre of the orography.

6. Observation of mountain waves over Costa Rica

As have been shown in the previous pages, when an fluid moves over irregular terrain, the vertical velocity of the fluid at the interface will be upward or downward, depending on the horizontal direction of the fluid relative to the slope of the terrain. Due to the fact that the air is a continuous medium the vertical motion at the bottom will be felt through some depth extending above the lower boundary. Clouds can form if the air forced over the terrain is sufficiently moist.

Clouds can form in the upward motion areas of the mountain waves. Non precipitating clouds that form in the moist layers in direct response to the wave motions induced by flow over topography are referred to as waves clouds. According to Houze (1993), precipitating clouds can also be formed or enhanced by upslope motions and dried out by downslope motions, while the cumulonimbus clouds may be triggered in several ways by flow over terrain.

As remarked by Scorer and Verkaik (1989) a surprising feature of lee waves is that the patterns are always much less complicated than the mountain shapes which produce them. Pilots with passengers prefer to avoid such waves because they alter the height of the plane and often contain regions of turbulence

The availability of satellite imagery over the Central American region has made possible the detection of mountain wave cloud formation. The signature of mountain waves in a satellite photograph consists of a series of white and black lines which represent respectively areas where air ascends and descends. In the particular case shown here mountain waves are seen over the northwestern part of Costa Rica, namely over the province of Guanacaste. The waves have developed on the lee side of the northern tip of the mountain range.

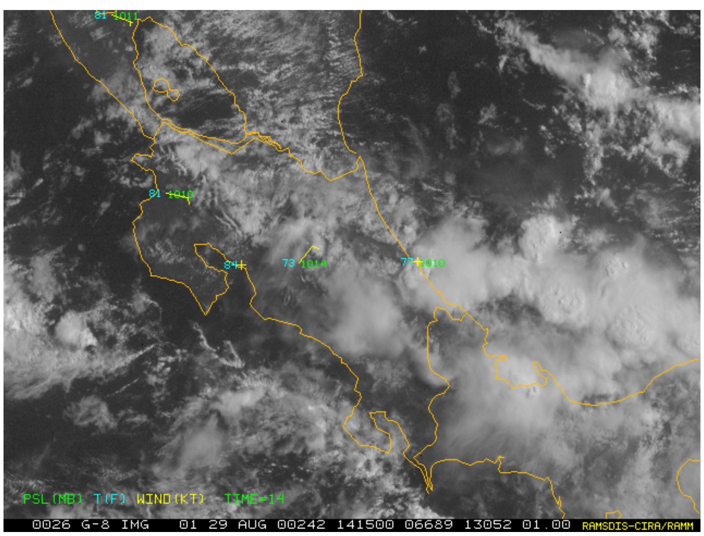

Figure 6: Satellite image from GOES 8. 24 August 2000 at 14:15 local time.

7. Conclusions

The interaction of a high Froude number flow with a mesoscale mountain range generates mountain waves that propagate in the vertical with phase lines that are tilted in the direction of the incoming flow.

The three dimensional nature of the flow is rather complex and there are clear differences between the flow close to the graphy and the flow further away from the orography.

Satellite imagery confirms that lee waves do occur in Costa Rica past mountain ranges.

The availability of a mountain wave forecast could allow pilots about the possibility of clear air turbulence.

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