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Development of a dynamic pressure standard of low amplitudes and frequencies.

Alberto Juan Díaz Tey¹, Georges Govaere Vicarioli²

¹ Escuela de Ingeniería Industrial, Universidad de Costa Rica, San José, Costa Rica.

² Unidad de Ingeniería Marítima de Ríos y Estuarios, Instituto de Investigaciones en Ingeniería, Universidad de Costa Rica, San José, Costa Rica.

E-mail: alberto.diaztey@ucr.ac.cr

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Abstract.

The height of the gravitational wave is an influential magnitude in the estimation of its energy content, a very important parameter in the design of maritime structures such as piers and breakwaters, however, there are reasonable doubts among researchers regarding the quality of its measurement.

The objective of the present work was to demonstrate that a stationary calibrated pressure transmitter allows the indirect calculation of the wave height with satisfactory accuracy despite the fact that it is a dynamic event as long as its frequency range is low, as is the case with the gravitational waves of the Costa Rican Central Pacific Ocean: between 0.05 Hz and 0.39 Hz.

In the absence of a primary standard of periodic pressure disturbances, an alternate path was developed based on the characterization of the parameters of the differential equation characteristic of a pressure measurement system in a shock tube prototype from normal shock wave theory and subsequent verification in a periodic disturbance generator that its attenuation and delay are practically negligible at the frequency of 0,1 Hz.

The effect of the different geometry of the chamber of the pressure measurement system used in the sea was evaluated pneumatically by comparison with the dynamic pressure standard in the prototype of the periodic disturbance generator, while the effect of the compressibility of the fluid was evaluated in water in the wave channel of the Laboratory of Maritime Engineering, Rivers and Estuaries of the University of Costa Rica.

The results show that the pressure measured from the least-squared adjustment coefficients of an electric current transmitter obtained by stationary calibration is acceptable to estimate the climatology of the gravitational wave characteristic of the Costa Rican Central Pacific Ocean with maximum errors of 136 mm in waves of height up to 1,4 m.

Keywords: dynamic calibration, pressure transducers, shock tube, periodic pressure generator.

1 Introduction.

Currently, the applications of measurements in dynamic processes are increasingly relevant, covering different fields in the technological, medical and academic sectors [1].

This work shows a very specific application related to the measurement of the height of the gravitational wave, a dynamic event characterized by waves with periods from 1 to 30 s and heights up to 15 m [2]. The energy content of the gravitational wave is transferred by the wind and its estimation from the height of the wave defines the design of maritime structures such as marinas and breakwaters.

Fortunately, the gravitational wave on the coasts of the Costa Rican central Pacific Ocean is a dynamic event characterized by

waves of low height and frequency: between 0,29 and 4,22 m and between 0,39 and 0,05 Hz respectively [3], by the "shield cone" effect that Cocos Island produces to gravitational wave trains formed in the far south Pacific Ocean near New Zealand.

The wave height is calculated indirectly by the pressure of the seawater column according to the Bernoulli's linearized equation [2], which for the case of interest represents between 2,81 and 42,31 kPa.

To measure the equivalent pressure of the wave height in the project range, it's possible to use the measurement system [4] GW WLI6U-030-050 [5] composed of a measurement chamber and a data acquisition system (DAQ) with a low sampling frequency $f_m \leq 10$ Hz.

The measurement chamber is divided into two parts: one is a metallic resistive sensor on a silicon diaphragm exposed to the hydrostatic pressure of the seawater column, electrically connected to a conditioning circuit of the analog electrical signal with a transmitter of 4 to 20 mA DC electric current located in the other hermetic part of the measuring chamber.

The transmitter cables are connected to the DAQ by a flexible hollow tube of variable length depending on the model, which also allows compensation of the measurements by atmospheric pressure.

Two batteries allow the autonomy of the measurement system until the programmed download of the information through the communication port and the software to the personal computer (PC).

Figure 1 shows the simplified representation of both the gravitational wave (oscillating line in blue) of constant period (T_p) as well as the installation of the GW WLI6U-030-050 measurement system on the seabed measure the wave height:

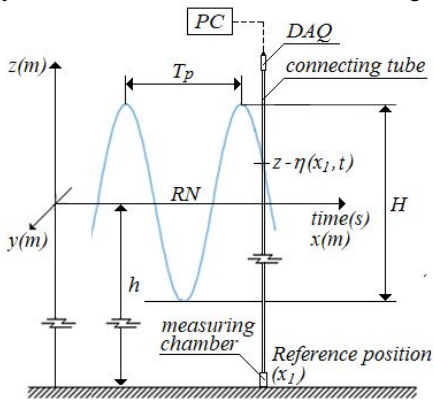


Figure 1. Installation of the measurement system at sea for a reference position

Although the GW WLI6U-030-050 [5] measurement system is used in a dynamic environment, its analog electrical output is converted into gauge pressure by means of a line of coefficients adjusted by the ordinary least-squares (OLS) method in a direct comparison with a standard pressure in an isobaric environment.

This contradictory situation between the stationary calibration of the measurement system and its application in a dynamic event caused a reasonable doubt among the researchers of the Laboratory of Maritime Engineering, Rivers and Estuaries (IMARES) of the UCR, for which it was proposed to demonstrate that the stationary calibration is suitable for measuring the height of the gravitational wave as a dynamic event in the range of the gravitational wave climatology of the Central Costa Rican Ocean [3].

For this purpose, their corrected response under stationary conditions was compared with the response of a dynamic pressure standard previously characterized in a shock tube, when both were exposed to periodic disturbances reproduced experimentally in a pneumatic generator.

At the end of the process, its corrected response under stationary conditions was compared with the response of the dynamic pressure standard installed in the IMARES wave channel.

2 Materials and methods.

2.1 Measurement of fluid pressure.

Considering that the response p_f (hPa) of the pressure measurement system in a fluid at rest is linear:

$$p_f = S_0 + S_1 i_s \quad (1)$$

Where i_s (mA) is the pressure transmitter output DC electric current, S_0 (hPa) is the adjusted intercept and S_1 (hPa/mA) is the static sensitivity coefficient.

Then the coefficients of equation (1) are estimated by the ordinary least-squares (OLS) method by direct comparison with a stationary pressure standard that defines its metrological traceability to SI [4].

$$(S_0, S_1) \rightarrow (\hat{S}_0, \hat{S}_1)$$

The comparison is made in an isobaric comparator with adjustable pressure in the range of interest: up to 42,31 kPa, taking care that the standard and calibrating sensors are in the same plane and depth of the working volume.

If the quality of the calibrating response is linearly acceptable then the fluid pressure is indirectly measured according to the following statistical model:

$$\hat{p}_f = \hat{S}_0 + \hat{S}_1 i_m \quad (2)$$

Where \hat{p}_f (hPa) is the measured pressure of the fluid and i_m (mA) is the output of the measured pressure transmitter. In this case, the measurement system behaves like a zero-order system and is the simplest dynamically.

However, when the pressure measurement system is used to measure dynamic events such as sea waves, then the parameters set in steady state are not in principle adequate to estimate the pressure.

2.2 Response to a periodic disturbance.

If that same pressure measurement system is disturbed by a perfect sinusoidal signal of amplitude A_p :

$$p(t) = A_p \sin(\omega t) \quad (3)$$

Being ω (rad/s) the frequency of the periodic disturbance and t is the time (s), then its response in the time domain is also a sinusoidal periodic function but modified in amplitude and phase with respect to the disturbance [6][7]:

$$p_f(t) = A_p M [\sin(\omega t + \phi)] \quad (4)$$

Where M (dimensionless) is the amplification and ϕ (rad) the phase shift of the response of the pressure measurement system.

The amplification of the response of the pressure measurement system is calculated according to the following equation [6][7]:

$$M = [(1 - a^2)^2 + (2\zeta a)^2]^{-0.5} \quad (5)$$

Where ζ (dimensionless) is the relative damping factor and a is an auxiliary variable:

$$a = \omega \times \omega_n^{-1} \quad (6)$$

Where ω_n (rad/s) is the natural frequency of the pressure measurement system.

The offset of the response of the pressure measurement system is calculated according to the following equation [6][7]:

$$\phi = -\tan^{-1} [(2\zeta a)(1 - a^2)^{-1}] \quad (7)$$

The phase shift between the periodic disturbance and the response of the pressure measurement system ($\Delta\tau$) can be expressed in the time domain according to the following equation:

$$\Delta\tau = 1000 \frac{ms}{s} (\phi \omega^{-1}) \quad (8)$$

Therefore, the challenge is to determine the parameters ζ and ω_n .

2.3 Response to an aperiodic shock.

If the sensor is exposed to an aperiodic, step-type disturbance of amplitude A_p :

$$p(t) = \begin{cases} 0 & \forall t < 0 \\ A_p & \forall t \geq 0 \end{cases} \quad (10)$$

The response in time ($\forall t \geq 0$) corresponds to an underdamped second order measurement system ($\zeta < 1$) [6][7]:

$$p_f(t) = A_p \left[1 - \frac{\text{EXP}(-\omega_d t)}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \cos^{-1}\zeta) \right] \quad (11)$$

Where ω_d (rad/s) is the damped frequency.

$$\omega_d = \sqrt{1-\zeta^2} \omega_n \quad (12)$$

Once the parameters of the characteristic equation $\hat{\omega}_n$ and $\hat{\zeta}$ of the dynamic pressure standard have been calculated with the established experimental accuracy, then in principle, it's possible to predict their behavior in the face of a periodic disturbance.

If the disturbance to which it's subjected is sinusoidal, its response will have the same shape, although the amplitude and phase may be remarkably different with respect to those of the disturbance.

2.4 Reconstruction of the response according to Fourier.

However, a periodic signal doesn't necessarily have to be sinusoidal to evaluate the dynamic behavior of a pressure transducer, and in experimentation, it's practically impossible to reproduce it.

In general, a periodic half-period function L_p can be "reconstructed" from its corresponding Fourier series if the Dirichlet condition [7] is fulfilled:

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t) \quad (13)$$

Where $n = 1, 2, \dots, \infty$

$$a_0 = \frac{1}{L_p} \int_{-L_p}^{L_p} f(t) dt$$

$$a_n = \frac{1}{L_p} \int_{-L_p}^{L_p} f(t) \cos(n\omega t) dt$$

$$L_p = 0,5 T_p$$

The amplitude and frequency of the harmonics of the reconstructed signal are calculated according to the following equations:

$$A_n = [a_n^2 + b_n^2]^{0,5} \quad (14)$$

$$\omega_n = n\omega_0 \quad (15)$$

Being $\omega_1 = \omega_0$ (rad/s) the fundamental frequency.

Since the mean is not an is-biased estimator for a periodic function, it's preferable to use the root mean square value:

$$\text{RMS} = \left[\left(\frac{a_0}{2} \right)^2 + \sum_{n=1}^{\infty} \left(\frac{A_n}{\sqrt{2}} \right)^2 \right]^{0,5} \quad (16)$$

Where $n = 1, 2, \dots, \infty$

$a_0/2$ (dimensionless) is the direct component of the signal.

2.5 The shock tube.

The shock tube [6][7][8] is an aperiodic step-type pressure disturbance generator, suitable for estimating the parameters ζ and ω_n of the pressure measurement system.

The prototype developed (Figure 2) is made up of two schedule 80 [9] PVC chambers of constant cross section (76,2 mm): the compression chamber of length $L_{cc} = 423,00(0,30) \text{ mm}$ and the chamber expansion of length $L_{ce} = 2\,233,40(0,51) \text{ mm}$, between which an aluminum diaphragm (6) of defined thickness is installed.

The two PVC tubular chambers are supported by supports (1) to a horizontal table (2) that ensures the stability of the system's operation. Each section is joined by movable PVC flanges (3), which guarantee airtightness by means of their couplings with rubber gaskets and four suitably adjusted bolts.

The inlet of humid air from a compressor is connected by hose and quick coupling (4) to the compression chamber and is controlled by the regulation valve (5) which, when abruptly opens, causes the necessary overpressure for the diaphragm (6) to burst.

If the air in the expansion chamber behaves like an ideal gas, it's possible to theoretically predict [6][7][8] the absolute pressure of the air disturbed by the first reflection of the shock wave P_{5t} (hPa):

$$P_{5t} = P_{2t} \frac{\left(\frac{\gamma_1 + 1}{\gamma_1 - 1} + 2 \right) P_{2t} P_{1e}^{-1} - 1}{\frac{\gamma_1 + 1}{\gamma_1 - 1} + P_{2t} P_{1e}^{-1}} \quad (17)$$

Where P_{2t} (hPa) is the theoretical absolute pressure of the disturbed gas at position TP2-1, γ_1 (dimensionless) is the isentropic coefficient of air and P_{1e} (hPa) is the absolute pressure of the gas at rest.

The absolute pressure P_{2t} is calculated according to [6][7][8]:

$$P_{2t} = P_{1e} \left[1 + \frac{2\gamma_1}{\gamma_1 + 1} (M_s^2 - 1) \right] \quad (18)$$

Where M_s is the Mach number (dimensionless):

$$M_s = a_s a_1^{-1} \quad (19)$$

The speed of the shock wave a_s (m/s) is measured with two OMEGA PX4200-030G [10] gauge pressure transmitters installed in the measurement ports TP2-1 and TP2-2, which detect the passage of the front of the shock wave in time:

$$a_s = L_2 \tau_2^{-1} \quad (20)$$

Where τ_2 is the time taken by the front of the shock wave to travel the distance $L_{21} = 1\,194,50(0,42) \text{ mm}$.

The speed of sound a_1 (m/s) at the thermodynamic conditions of the air at rest is calculated according to [6][7][8]:

$$a_1 = (\gamma_1 R T_1 M_g^{-1})^{0,5} \quad (21)$$

Where $R = 8,314\,462\,1(75) \text{ (J/mol K)}$ [11] is the molar constant of the gas, T_1 (K) is the thermodynamic temperature and $M_{as} = 0,289\,7(4,5 \times 10^{-7}) \text{ g} \cdot \text{mol}^{-1}$ is the molar mass of dry air [12].

The thermodynamic temperature of the air at rest T_1 (K) trapped in the expansion chamber is estimated assuming the thermal equilibrium with the ambient air. Ambient air conditions: temperature, humidity and atmospheric pressure are continuously measured and recorded with the DAQ EXTECH SD700 s/n Q774023 [13].

The still air gauge pressure is measured with the OMEGA PX4200-030G [10] s/n 56901 gauge pressure transmitter installed in the TP2-1 measurement port.

The still air gauge pressure is measured with the OMEGA PX4200-030G [10] s/n 56901 gauge pressure transmitter installed in the TP2-1 measurement port.

The referred measurement systems have been calibrated at the INM of Costa Rica in a stationary regime, thus guaranteeing metrological traceability to the SI.

2.6 Measurement of pressure transmitters output.

The DC electric current from the output of the pressure transmitters [i_m in equation (2)] is measured independently, simultaneously and synchronized by each of the four input channels of the milliammeter of the HBM QuantumX MX440B data acquisition system [14] s/n 0009E5008567.

The milliammeter calibration of the data acquisition system is calibrated in steady state by direct comparison with DC electrical current signals from a FLUKE 753 [15] reference source, calibrated in a Laboratory authorized by the manufacturer.

2.7 Drying the transmission gas.

The means of displacement of the shock wave is the ambient air trapped in the expansion chamber during the assembly of the diaphragm but dried with dry air for medicinal use (purity $\approx 98\%$), which is supplied from its respective container to the expansion chamber through valve V1 and is extracted by overpressure with respect to atmospheric with valve V2 (Figure 2).

The reduction of air humidity is demonstrated by introducing the datalogger HOBO U12-012 [16] 160 mm into the expansion chamber with a sampling frequency of 10 Hz (Figure 3).

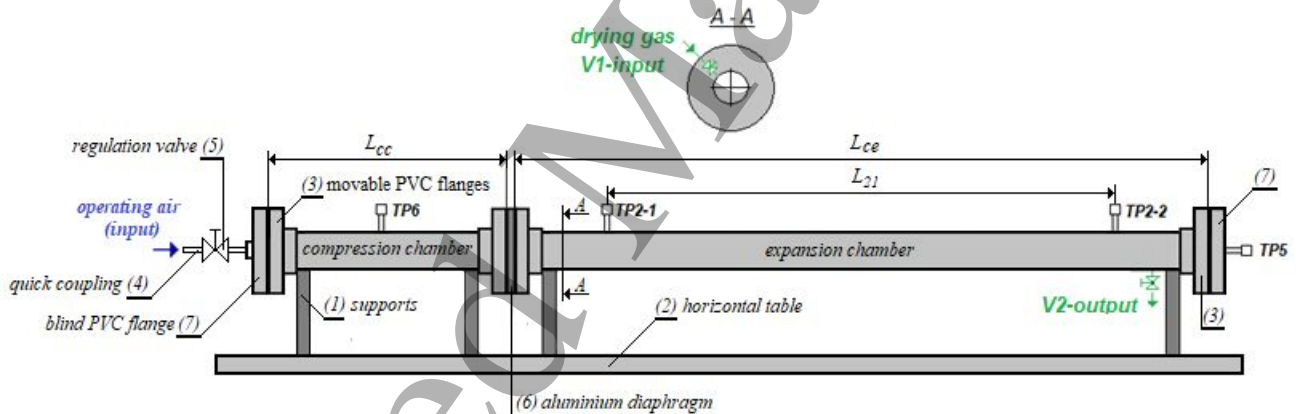


Figure 2. General schematic of the prototype shock tube TC-02-(0.6/2).

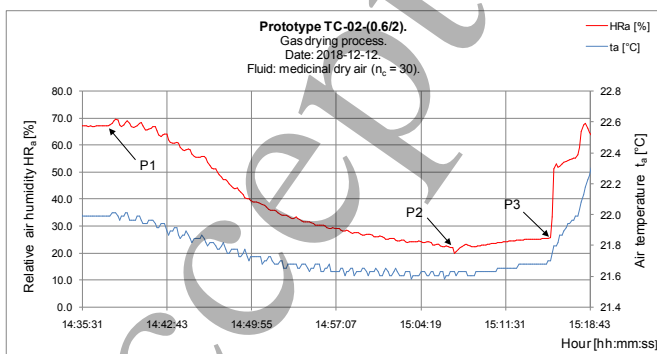


Figure 3. Air drying process in the expansion chamber.

The drying of the atmospheric air trapped in the expansion chamber is a continuous process of $n_c = 30$ fill-empty cycles with the medicinal dry air.

The drying starts at point P1 and point P2 ends with an effective reduction of the relative humidity up to 25% in 30 min before performing the "shot" at point P3.

Although the pressure of the gas disturbed by the first reflection of the shock wave remains constant, the duration of the event is so short that it could not be detected by the data acquisition system of the GW WLI6U-030-050 [5].

For this reason, the pressure transmitter EXTECH PT30-SD [17] s/n TA87982 was dynamically characterized with a sensor of the same nature and although the measurement chamber is smaller, it was considered that the resonant effect is similar.

The gauge pressure measurement system consisting of the EXTECH PT30-SD pressure transmitter [17] and the HBM QuantumX MX440B data acquisition system [14] was considered the dynamic pressure standard of this project.

3 Results and discussion.

3.1 Validation of the shock tube prototype.

The validation [4] of the TC-02-(0.6/2) shock tube prototype was based on the measurement of its performance parameters when a 10,2 μm thick aluminum diaphragm bursts and its comparison with those previously obtained in the TC10 shock tube of the Laboratory of Dynamic Measurements of the École Nationale Supérieure d'Arts et Métiers (LMD-ENSAM) [18][19], with the expectation of 5% of difference.



Figure 4. Prototype TC-02-(0.6/2) built.

The validation [4] of the TC-02-(0.6/2) shock tube was based on the measurement of its performance parameters when a 10,2 μm thick aluminum diaphragm bursts and its comparison with those previously obtained in the TC10 shock tube of the Laboratory of Dynamic Measurements of the École Nationale Supérieure d'Arts et Métiers (LMD-ENSAM) [18][19], with the expectation of 5% of difference.

The first performance parameter is the rise time of the air pressure at rest from its perturbation by the first reflection of the shock wave ($\Delta\tau_{15}$) and is defined by the milestones P1 (shock wave front impact) and P2 (response to disturbance of the sensor) of the EXTECH PT30-SD pressure transmitter installed in the measurement port TP5 (see Figure 2).

In principle $\Delta\tau_{15} \rightarrow 0$ but the imperfection of its reproduction due to the nature of the dynamic pressure standard prevents its achievement.

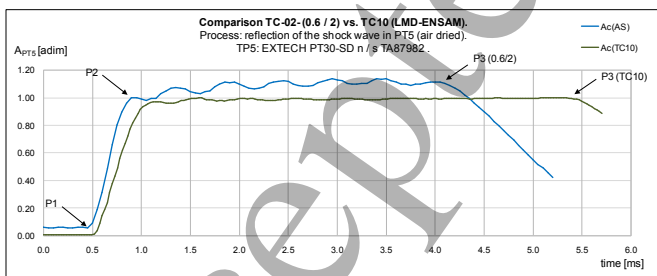


Figure 5. Pressure of the disturbed gas in the expansion chamber.

The second performance parameter is the pressure stabilization time ($\Delta\tau_5$) defined by milestones P3 (the arrival of the rarefaction wave coming from the compression chamber of the shock tube causes the pressure drop) and P2.

In principle $\Delta\tau_5 \rightarrow \infty$ but the effect of the successive reflections of the expansion and rarefaction shock waves [7] shortens its constancy.

Table 1 shows the performance of the TC-02-(0.6/2) prototype with respect to the French TC10, measured with the dynamic pressure standard for a sampling frequency equal to 20 kHz:

Table 1. Comparison of performance parameters.

Parameters	MU	TC-02-(0.6/2) (AS)			TC10		
		y	$s(y)$	ν	y	$s(y)$	ν
$\Delta\tau_{15}$	ms	0.850	0.000	3	0.638	0.018	1
$\Delta\tau_5$	ms	2.700	0.065	3	4.232	0.092	1

Where $\nu = n_d - 1$ is the degrees of freedom and ν is the number of "shots fired".

The deviation from the supposed isentropic behavior of the shock wave expansion is estimated with the statistical criterion "trueness" [20]. Table 2 shows the results of the comparison of the "trueness" of the pressures measured in the measurement ports TP2-1 and TP5 of the shock tube prototype.

Table 2. Trueness of measured pressures.

Parameters	MU	TP2-1			TP5		
		y	$u_r(y)$	v_{ef}	y	$u_r(y)$	v_{ef}
P_{Ic}	hPa	887,27	0,13%	14	882,87	0,17%	12
P_{e-TP}	hPa	979,58	0,12%	14	1072,09	0,14%	20
P_{t-TP}	hPa	962,37	1,00%	1842	1042,86	1,40%	3684
$\delta(P_{TP})$	%	1,79%			2,80%		

Where $u_r(y)$ is the relative uncertainty of the parameter y estimated according to the Law of propagation of uncertainties [21] and v_{ef} the effective degrees of freedom according to Welch-Satterthwaite [21].

The responses of the corrected dynamic standard in both shock tubes: TC-02-(06/2) (in blue) and TC10 (in green), correspond to those of an underdamped second-order measurement system ($\zeta < 1$) according to equation (11) and although there are different methods to adjust the coefficients, the Levenberg-Marquardt numerical method was used, based on an iterative algorithm of approximations that minimizes the residual of the adjustment.

Table 3. Comparison of the adjusted parameters.

Parameters	Symbols	MU	$y_{(TC-02)}$	$y_{(TC10)}$
Relative damping coefficient	ζ_n	dm	0,7923	0.7668
Damped frequency	ω_d	rad/ms	4,7013	4,9395
Angular natural frequency	ω_n	rad/ms	7,7058	7,6954
Linear natural frequency	f_n	kHz	1,2264	1,2248
Adjustment error	s_a	adim	0,0904	0,0426
	v_a	sample	59	186

The maximum relative error of the adjusted coefficients by the Levenberg-Marquardt numerical method in both shock tubes didn't exceed the $4,82\% < 5\%$ established as a goal.

3.2 The generator of periodic pressure disturbances.

In 1972 Hilton [7] built a periodic pressure generator that was essentially a tube partially filled with liquid, with one end open and the other closed, mounted on an electrodynamic stirrer, to obtain a sinusoidal disturbance.

Based on the results obtained, Hilton [7] recommended abandoning the primary nature of the method and installing a reference pressure transducer alongside the calibrator.

In 1956 Hermann and Stiefelmeyer [7] built a periodic pressure generator based on a rotary valve that generated square signals slightly distorted by the effect of its sequential opening-closing.

More recently, in 2000, Kobata and Ooiwa [22][23] developed a generator of periodic pressure disturbances based on a rotary valve of a novel design compared to the one built by Hermann and Stiefelmeyer in 1956 [7] to calibrate transmitters of pressure. pressure in amplitudes up to 10 kPa and fundamental frequency between 10 and 50 Hz.

Like their predecessors, Kobata and Ooiwa [22][23] considered that a reference pressure transmitter is required for the dynamic calibration of another pressure transmitter.

3.3 The construction of the prototype.

In the same way that Hermann-Stiefelmeyer [7] and later Kobata and Ooiwa [22][23] predicted that a rotary valve would generate a train of square pulses, it was considered that the alternating and synchronized control of the inlet and outlet of the flow of air through valves could generate a train of triangular pulses, with a period and amplitude similar to those of the gravitational waves of the Costa Rican Central Pacific.

To this end, the prototype GPP-02-A (Figure 6) was built between June and October 2018, consisting of a schedule 40 [9] PVC measuring chamber of constant cross section (76,2 mm) and length $L_c = 423,00(0,30)$ mm.

In the measurement chamber there are two measurement ports: T_{Pp} and T_{Pc} displaced 90° in the same plane, where the dynamic pressure standard, previously characterized in the shock tube prototype, and the calibrating one are installed.

To experimentally reproduce the periodic pressure signal, the GPP-02-A prototype consists of two control systems: the first consisting of the fine adjustment valve (8) and the second consisting of a PLC that controls the solenoids of the ON-OFF valves normally open, installed at the inlet and outlet (positions 5e and 5s) of the measurement chamber.

To reproduce the pneumatic periodic signal with the desired parameters, the following procedure is carried out:

- The operating gas supply source is connected to the inlet of the fine adjustment ON-OFF valve (8) and with the second control system disconnected, the flow is adjusted by carefully opening the fine adjustment ON-OFF valve (8) up to the desired initial operating pressure (p_{op}).
- The initial operating pressure is adjusted by measuring in real time on the PC screen the pressure of the dynamic pressure standard installed in the measurement port TPp.

- Once the initial pressure has stabilized at the desired value, the operating period and the number of cycles are programmed in the PLC, activating its control routine until the final amplitude of the reproduced periodic signal is reached.
- Simultaneously, the recording of the measurements is activated and once the programmed cycles are concluded, the results are downloaded for further processing. Without altering the configuration of the pneumatic circuit, the process is repeated under conditions of intermediate precision [20] as many times as deemed appropriate.

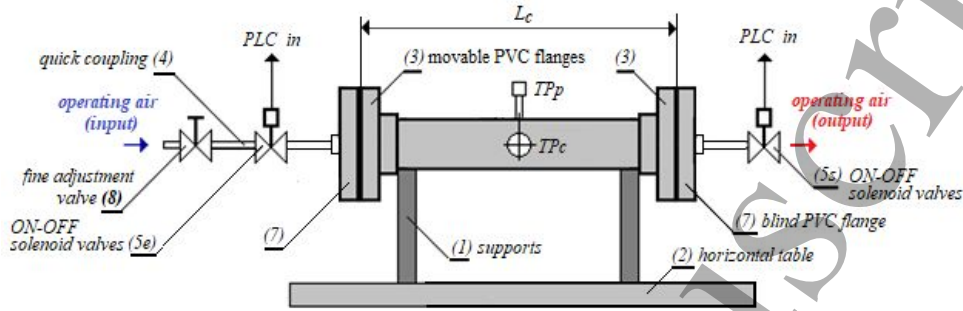


Figure 6. General diagram of the prototype of the generator of periodic disturbances GPP-02-A.

3.4 Validation of the prototype.

The characteristic parameters of the periodic pneumatic signals reproduced in the GDP-02-A prototype are peak-peak period T_p (s) and amplitude A_p (hPa):

$$A_p = (p_{c,max} - p_{c,min}) \quad (22)$$

Where $p_{c,max}$ (hPa) and $p_{c,min}$ (hPa) are the maximum and minimum corrected gauge pressures of the cycle.

The results of a run composed of five reproduced cycles are shown in the following figure:

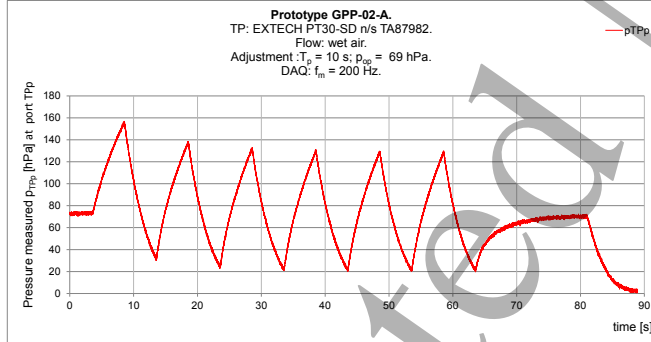


Figure 7. Reproduction of the periodic signal ($T_p = 10$ s).

The average results obtained were the following:

$$T_p = (10,030 \pm_{-0,070}^{+0,080}) s$$

$$A_p = (71,550 \pm_{-2,262}^{+4,163}) hPa$$

The initial assumption of explaining the nature of the signal generated by a triangular pulse was not fulfilled, but considering that the signal is periodic and satisfies the Dirichlet condition, it's "reconstructed" with a Fourier series:

- The experimental distribution of the signal on each side of the half-period was adjusted by linear regression using a polynomial of degree 3, in this case $L_p = 5,015$ s.

Table 4. Results of ordinary least-squares adjustment.

Ranges	C_0	C_1	C_2	C_3	$s_a(A_p)$
$0 \leq t < L_p$	0,02616	0.3334	-4599E-5	367E-5	40E-5
$L_p \leq t < 2L_p$	4,080	-0.9192	7108E-5	-198E-5	314E-5

Where $R^2(A_p)$ is the multiple correlation coefficient equal to 1,000 00 for both cases.

- $F_n(t)$ is calculated according to equation (11). In this case, only harmonics were taken into account up to $n = 13$, that is, $F_{13}(t)$.
- The terms and parameters of the Fourier series are calculated according to the equations in section 2.4:

Table 5. Fourier series terms and parameters.

Parameters	Symbols	MU	Values
Fundamental frequency	$\omega_{n=1}$	rad/s	0,6264
Direct component	$A_{n=0}$	dm	0,9832
Amplitude of the fundamental harmonic	$A_{n=1}$	dm	0,1701
Total harmonic distortion	THD	%	34,16
Root mean square	RMS	dm	0,5102

The superposition of the measurements (red line) together with the curves fitted by the ordinary least-square method (blue line) and reconstructed by Fourier series (green line) demonstrate the agreement of the two approaches to explain the periodic event.

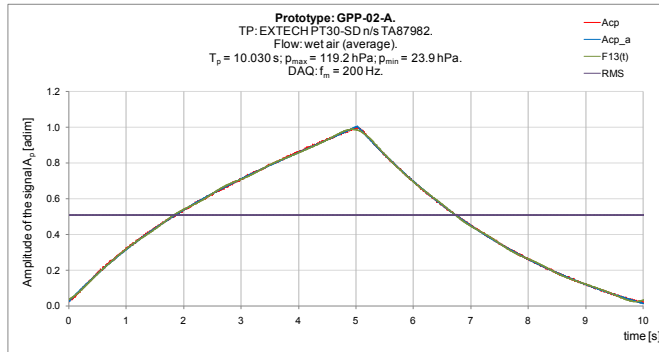


Figure 8. Comparison of the experimental and adjusted signals.

Where A_p (dimensionless) is the amplitude of the signal but referred to the maximum pressure of the cycle.

The frequency and amplitude spectra obtained from the processing of the average run are shown in Table 6:

Table 6. Frequency and amplitude spectra (first part).

ω_n [rad/s]						
0	1	2	3	4	5	6
0	0.6264	1.2529	1.8793	2.5058	3.1322	3.7586
A_n [dm]						
0.9832	0.1701	0.0002	0.0027	0.0000	0.0004	0.0000

Table 6. Part two (continued).

ω_n [rad/s]						
7	8	9	10	11	12	13
4.3851	5.0115	5.6380	6.2644	6.8908	7.5173	8.1437
A_n [dm]						
0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

The spectra obtained show that the effect of frequency on the amplitude of the harmonics of the reproduced periodic signal is practically zero from the second harmonic. In this sense, the observed distortion estimated by the total harmonic distortion (THD = 34.16%) is attributable to the non-linear behavior of the opening-closing valves in the GPP-02-A prototype.

In this regard, Kobata and Ooiwa [22][23] reported that in their periodic generator the assignable cause was the resonant effect of the chamber on the rotary valve.

3.5 Applications.

Considering that the geometry of the measurement chamber and the low sampling frequency of the GW WLI6U-030-050 [5] measurement system prevent its dynamic calibration in the prototype shock tube TC-02-(0.6/2), verified the hypothesis: "the least-squared coefficients estimated in a stationary calibration allow the indirect calculation of the wave height with adequate accuracy in the range of low amplitude and frequency typical of the Costa Rican Central Pacific Ocean."

For this purpose, the GW WLI6U-030-050 [5] s/n 1604000374, hereinafter referred to as calibrant, was selected and installed in the TPC measurement port of the GPP-02-A prototype:

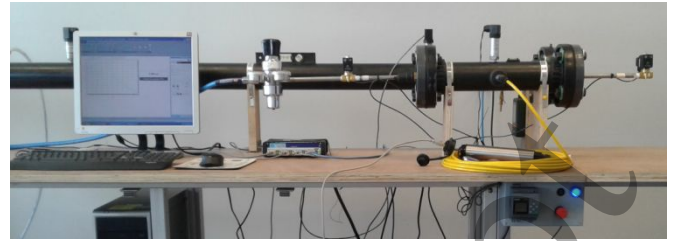


Figure 9. Installation of the GW WLI6U-030-050 on the GPP-02-A.

The verification of the hypothesis was carried out through the stationary calibration of the measurand and subsequent verification of its metrological performance in the face of a reproduced periodic disturbance.

3.6 The stationary calibration of the calibrant.

The stationary calibration of the calibrant was based on the estimation of its least-squares coefficients by direct comparison with the stationary pressure standard OMEGA PX4200-030G [10] s/n 56901 calibrated in the Pressure Laboratory of the INM of Costa Rica, adjusting the operation of the prototype GPP-02-A for its operation as an isobaric comparison medium.

Table 7. OLS fit coefficients.

S_0 [hPa]	$s(S_0)$ [hPa]	S_1 [hPa/mA]	$s(S_1)$ [hPa/mA]	R^2 [dm]	$s(pc_{aj})$ [hPa]
-229.035	1.7554	58.440	0.1508	0.99998	1.774

3.7 The dynamic verification of the calibrant.

The first verification of the metrological performance of the calibrant was carried out with humid air in the GPP-02-A prototype, using as references the stationary and dynamically corrected responses of the dynamic pressure standard.

Considering that the period of the disturbance reproduced in the GPP-02-A prototype is $T_p = 10,015$ s then the response of the dynamic pressure standard is defined by the following parameters:

Table 8. Parameters of the response to periodic disturbance.

Parameters	Symbols	MU	Values
Auxiliary variable	a	dm	0.0001
Attenuation	M	dm	1.0000
Phase shift	ϕ	rad	-0.0001
Phase shift in time domain	$\Delta\tau_{(average)}$	μ s	-206

From Table 8 it is concluded that the response of the dynamic pressure standard is practically not affected by the disturbance: there is no attenuation ($M = 1$) and the delay is negligible (206 μ m), therefore, the pneumatic signal is a reference reliable to evaluate the response of the calibrant.

Considering that local acceleration of gravity $g_l = 9.7818$ m s⁻² and the density of water $\rho_w = 997,52$ kg m⁻³, then the comparative results in equivalent water column height (H), calculated from Bernoulli's linearized equation is shown in Figure 10:

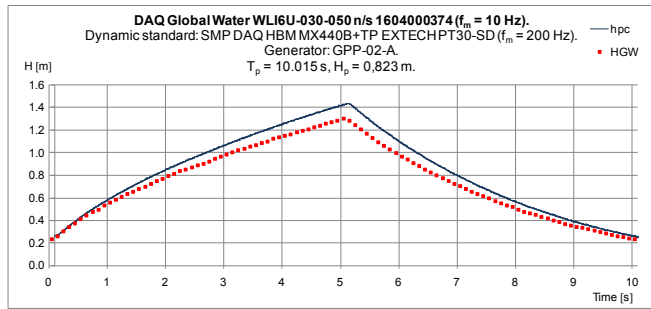


Figure 10. Comparative responses to a pneumatic disturbance.

3.8 IMARES wave channel.

The second verification of the metrological performance of the calibrant was carried out with water in the IMARES wave channel, using as a reference the installed pressure measurement system based on the electrical conductivity of the fluid.



Figure 11. IMARES wave channel.

In this case, the calibrant was submerged 1 m from the free surface to avoid the effect of the water column when the wave passed [2] and it was subjected to regular periodic disturbances of period $T_p = 1.8$ s.

Being the same density of the water in the wave channel, the comparative results in equivalent water column height, calculated from Bernoulli is shown in Figure 12:

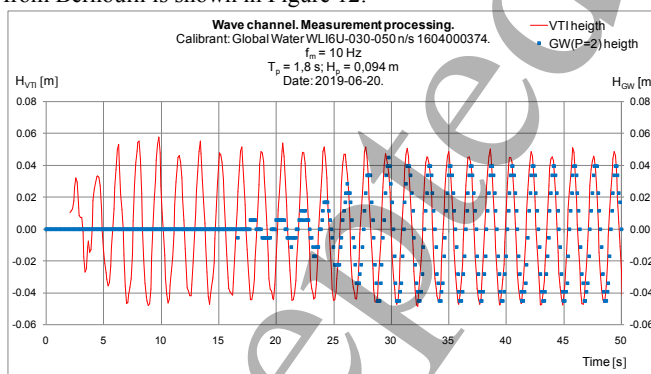


Figure 12. Comparison of the height measured in the wave channel.

From the swarm of measurements, those corresponding to the four cycles were averaged between 30,5 s and 37,9 s:

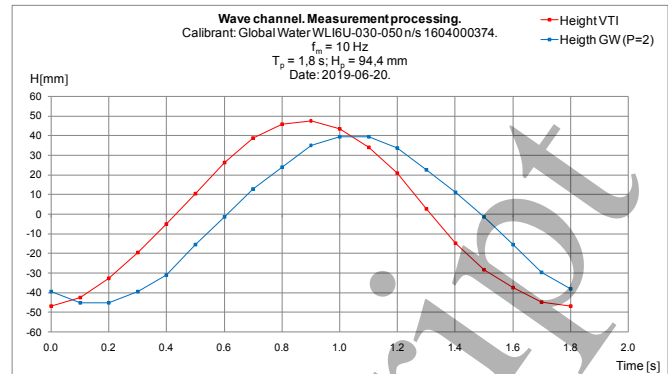


Figure 13. Comparative responses to a hydraulic disturbance.

The comparative results of the measurements in column of air and equivalents in water according to Bernoulli are shown in Table 9:

Table 9. Head pressure comparison.

Statistics	GPP-02-A (air)			Wave channel (water)		
	H_p	H_{GW}	ΔH	H_p	H_{GW}	ΔH
	m	m	mm	m	m	mm
Minimum	0.253	0.231	-21.3 (-8.4%)	0.092	0.084	-7.5 (-8.2%)
Average	0.823	0.747	-76.6 (-9.3%)	0.094	0.084	-9.9 (-10.5%)
Maximum	1.435	1.299	-135.9 (-9.5%)	0.098	0.084	-14.0 (-14.2%)

4 Conclusions.

From Table 9 it can be seen that the relative errors of the measurements of the wave heights reproduced in air in the generator of periodic disturbances GPP-02-A and in water in the wave channel of the CIMAR-UCR are of the same order: - 9.3% vs. - 10.5% despite the fact that the pressure measurement systems and the frequencies of the events were different, demonstrating the robustness of the approach.

Therefore, the height of the gravitational wave as a periodic low frequency event can be measured with a measurement system of the type GW WLI6U-030-050 [5] calibrated in steady state with maximum errors of 136 mm in waves of height up to 1,4 m.

As the frequency of the disturbance is not relevant to the established scope, it is appropriate to use the conventional theoretical framework of the GUM to calculate the uncertainty of the wave height, but taking into account that as it is an oscillating event, parametric statistics cannot be applied [24] to estimate the pooled variability under conditions of intermediate precision but if the arcsine distribution function [21].

The metrological traceability of the indirect water column height measurement systems is managed through the least-squared estimation of the adjustment coefficients in the stationary calibrations.

This property is of great importance when it is not possible to experimentally determine the coefficients of the characteristic equation of the pressure transmitter, either because it cannot be installed in the shock tube prototype or when the data acquisition system does not allow to increase the sampling frequency. to detect the milestones of the isentropic expansion of the gas.

Disclaimer.

The equipment or commercial instruments identified here are only to ensure the integrity of the information. Such identification does not imply recommendation or endorsement by the author, nor does it imply that the identified equipment is necessarily the best available for this purpose.

Data availability statement.

The data that support the findings of this study are available upon reasonable request from the author.

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ORCID iDs

Alberto Juan Díaz Tey <https://orcid.org/0000-0002-2294-7655>
Georges Govaere Vicarioli <https://orcid.org/0000-0002-0036-4571>

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