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ON THE STABILITY OF CERTAIN PERTURBED SYSTEMS OF DIFFERENTIAL EQUATIONS AND THE RELATIONSHIP WITH THE MAGNITUDE OF THE PERTURBATION

SOBRE LA ESTABILIDAD DE CIERTOS SISTEMAS PERTURBADOS DE ECUACIONES DIFERENCIALES Y SUS RELACIONES CON LA MAGNITUD DE LA PERTURBACIÓN

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Abstract

In this work we consider a class of polytopes of third order square matrices, studied early. We obtain a condition to guarantee Hurwitz stability of each of elements of the polytope. This condition is more simples than one obtained before. Taking into account that to the considered set of matrices correspond a family of perturbed systems of differential equations, we study the relationship between the stability condition and the magnitude of the class of perturbations considered for this family.

Resumen

En el presente trabajo consideramos una clase de politopos de matrices cuadradas de tercer orden, estudiada anteriormente. Obtenemos una condición para garantizar la estabilidad, según Hurwitz, de cada uno de los elementos del politopo. Dicha condición es más simple que la obtenida con anterioridad. Teniendo en cuenta que al conjunto considerado de matrices corresponde una familia de ecuaciones diferenciales perturbada, estudiamos la relación entre la condición de estabilidad y la magnitud de la clase de perturbaciones considerada para esta familia.

1 Introduction

In a previous article [7], we characterize Hurwitz–stability of the third order family of matrices, i.e., we study the stability properties of a convex and symmetric polytope of the time–invariant matrices. This set depends on a real positive parameter r, which variation represents a contraction or an expansion of the set. Besides, the considered set of matrices is an extension of the class of sets considered by Nicado & Hing (1998) in [5].

In this work, in the first section, we simplify the expressions for the calculation of the "extreme" value r now considering unused properties of the set of matrices, specifically the fact that the matrices which determine the polytope have only one linear independent row (or column). In the second section we analyze the relationship between optimum value of the parameter r and the magnitude of the perturbation. To the considered set corresponds a family of the perturbed systems of third order linear differential equations.

2 Formulation of the problem and calculation of the number $r^*(A,(B_i)_{i\in\overline{1,4}})$

Let be $A \in \mathbb{R}^{3\times 3}$ a Hurwitz–stable matrix, i.e., it has all their eigenvalues with negative real part; let be $B_i \in \mathbb{R}^{3\times 3}$, $i = \overline{1,4}$, matrices not all nulls and of the form $B_i = b(v^{(i)})^T$, $i = \overline{1,4}$, where $b \in \mathbb{R}^{3\times 1}$ is a constant vector, and the vectors $v^{(i)} \in \mathbb{R}^{3\times 1}$, $i = \overline{1,4}$, have the following coordinates:

$$\begin{aligned}(v^{(1)})^T &= (v_1^0, v_2^0, v_3^0)\\(v^{(2)})^T &= (-v_1^0, v_2^0, v_3^0)\\(v^{(3)})^T &= (v_1^0, -v_2^0, v_3^0)\\(v^{(4)})^T &= (v_1^0, v_2^0, -v_3^0),\end{aligned}$$

where v_q^0, $q = 1,2,3$, are given values.

We notate that for the matrices $B_i \in \mathbb{R}^{3\times 3}$, $i = \overline{1,4}$, it holds that $rk(B_i) = 1$, $i = \overline{1,4}$, where $rk(M)$ denotes the rank of matrix M.

For each number $r > 0$ we consider the convex and symmetric polytope depending on the parameter r and which is formed by time independent matrices: $\aleph(A,(B_i)_{i\in\underline{N}}, r) = conv\{A \pm rB_i, i \in \underline{N}\}$. The notation $conv\{\cdot\}$ means that the set is convex.

In general, when N matrices are considered, for the family $\aleph(A,(B_i)_{i\in\underline{N}}, r)$ we can formulate the problem as: Find the values of the $r > 0$ which guarantee the stability of the convex and symmetric polytope $\aleph(A,(B_i)_{i\in\underline{N}}, r)$; in other words, find the values of the parameter $r > 0$, so that each matrix $M \in \aleph(A,(B_i)_{i\in\underline{N}}, r)$ is stable. In this work we consider the particular case, when $N = 4$ and the four matrices $B_i \in \mathbb{R}^{3\times 3}$, $i = \overline{1,4}$, have rank equal to one.

In [7] we define the number $r^*(A,(B_i)_{i\in\overline{1,4}})$.

Definition 1 (Vázquez (2002)) *Let $A \in \mathbb{R}^{3\times 3}$ be a Hurwitz–stable matrix and $B_i \in \mathbb{R}^{3\times 3}$, $i = \overline{1,4}$, matrices no all null, then*

$$r^*(A,(B_i)_{i\in\overline{1,4}}) = \inf\Big\{r > 0/\aleph(A,(B_i)_{i\in\overline{1,4}}, r)$$

contains at least one non stable matrix.

Evidently, if we determine the number $r^*(A,(B_i)_{i\in\overline{1,4}})$, the stated problem will be solved, because the family of matrices $\aleph(A,(B_i)_{i\in\overline{1,4}}, r)$ is stable if and only if $r < r^*(A,(B_i)_{i\in\overline{1,4}})$.

We apply the Routh–Hurwitz theorem (see, for example, [4]) to the family of matrices $\aleph(A, (B_i)_{i\in\overline{1,4}}, r)$ in order to study the stability properties of this family. With that purpose we write the family $\aleph(A, (B_i)_{i\in\overline{1,4}}, r)$ in an equivalent form, which facilitate the application of the Routh–Hurwitz theorem. From this action we obtain three extreme problems and the following theorem.

Theorem 2 (Vázquez (2002)) *Let $A \in \mathbb{R}^{3\times 3}$ be a Hurwitz–stable matrix, let $B_i \in \mathbb{R}^{3\times 3}$, $i = \overline{1,4}$, be the matrices defined above, then the time invariant polytope $M(r)$ will be stable if and only if*

$$\begin{aligned} r &< \min\left\{\pi_1(A, (B_i)_{i\in\overline{1,4}}), \pi_2(A, (B_i)_{i\in\overline{1,4}}), \pi_3(A, (B_i)_{i\in\overline{1,4}})\right\} \\ &= r^*(A, (B_i)_{i\in\overline{1,4}}), \end{aligned}$$

where the numbers $\pi_1(A, (B_i)_{i\in\overline{1,4}})$, $\pi_2(A, (B_i)_{i\in\overline{1,4}})$ and $\pi_3(A, (B_i)_{i\in\overline{1,4}})$ are, respectively, the solutions of the following extreme problems:

$$\begin{array}{rll} i) & \sum_{j=1}^{8}\gamma_j \longrightarrow & \min \\ & s.t. & tr(A) + \xi^T\beta = 0, \gamma_j \geq 0, j = \overline{1,8} \\ ii) & \sum_{j=1}^{8}\gamma_j \longrightarrow & \min \\ & s.t. & \det A + \rho^T\beta = 0, \gamma_j \geq 0, j = \overline{1,8} \\ iii) & \sum_{j=1}^{8}\gamma_j \longrightarrow & \min \\ & s.t. & g + \langle\chi, \beta\rangle - \beta^T\eta\xi^T\beta = 0, \gamma_j \geq 0, j = \overline{1,8}, \end{array}$$

where $g = \det A - tr(A)\sum_{p=1}^{3}(A_{pp})$.

Proof. For the proof see Vázquez (2002). ■

Remark 1 *In [7] we define the polytope $M(r)$ as follows: With the matrices $A \in \mathbb{R}^{3\times 3}$ and $B_i \in \mathbb{R}^{3\times 3}$, $i = \overline{1,4}$, we form the matrices $M_{2i} = A - rB_i$; $M_{2i-1} = A + rB_i$; $i = \overline{1,4}$. Now we have the family $M(r) = conv\left\{M_j(r), j = \overline{1,8}\right\}$. Now, it is clear that $M(r) \equiv \aleph(A, (B_i)_{i\in\overline{1,4}}, r)$.*

Lemma 3 *Let $A \in \mathbb{R}^{3\times 3}$ and $B_i \in \mathbb{R}^{3\times 3}$, $i = \overline{1,4}$ matrices that satisfy the conditions of the theorem 2, then:*

$$\pi_1(A, (B_i)_{i\in\overline{1,4}}) = \frac{-trA}{\left|b_1 v_1^0\right|}\ ,\ \pi_2(A, (B_i)_{i\in\overline{1,4}}) = \frac{-\det A}{\left|\varpi_1 v_1^0\right| + \left|\varpi_2 v_2^0\right| + \left|\varpi_3 v_3^0\right|}$$

where

$$\varpi_1 = b_1\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33}\end{vmatrix}, \varpi_2 = b_1\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33}\end{vmatrix}, \varpi_3 = b_1\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32}\end{vmatrix}.$$

Proof. Evidently, problems i) and ii) are linear programming problems, so their solutions are achieved on the vertex $A \pm rB_i$, $i = \overline{1,4}$, of the polytope $M(r)$. This is equivalent to find the least positive root of the algebraic equations: $tr(A) \pm r\langle b, \nu^{(i)}\rangle = 0$, $\det(A) \pm r \left\langle \varpi, \nu^{(i)} \right\rangle = 0$, $i = \overline{1,4}$, i.e.,

$$\pi_1(A, (B_i)_{i\in\overline{1,4}}) = \inf \left\{ r > 0 / tr(A) + r \left| \langle b, \nu^{(i)} \rangle \right| = 0 \text{ for some } i = \overline{1,4} \right\},$$

$$\pi_2(A, (B_i)_{i\in\overline{1,4}}) = \inf \left\{ r > 0 / \det(A) + r \left| \langle \varpi, \nu^{(i)} \rangle \right| = 0 \text{ for some } i = \overline{1,4} \right\},$$

and the infimum is achieved for the greatest values of $\left|\langle b, \nu^{(i)}\rangle\right|$ and $\left|\langle \varpi, \nu^{(i)}\rangle\right|$ respectively, thus we obtain directly the statement of the lemma. ■

Lemma 4 *Let be $A \in \mathbb{R}^{3\times 3}$ and $B_i \in \mathbb{R}^{3\times 3}$, $i = \overline{1,4}$ matrices that satisfy the conditions of the theorem 2, then $\pi_3(A, (B_i)_{i\in\overline{1,4}}) = k^* U$, where U is the sum of the coordinates of the vector which is the solution of the following linear programming problem:*

$$\begin{cases} \sum_{j=1}^{8} u_j & \longrightarrow & \min \\ & s.t. & u_1 + \frac{1}{16\xi_1} \geq 0, u_2 + \frac{1}{16\xi_1} \geq 0 \\ & & u_3 + \frac{1}{16\xi_1} \geq 0, u_4 + \frac{1}{16\xi_1} \geq 0 \\ & & u_5 + \frac{1}{16\xi_1} \geq 0, u_6 + \frac{1}{16\xi_1} \geq 0 \\ & & u_7 + \frac{1}{16\xi_1} \geq 0, u_8 + \frac{1}{16\xi_1} \geq 0 \\ & & u_1 - u_2 - u_3 + u_4 + u_5 - u_6 + u_7 - u_8 = 0, \end{cases}$$

and k^* is the root of least absolute value of the equation $\Phi(k) = 0$, where

$$\begin{aligned} \Phi(k) \quad = \quad & -\frac{\eta_1}{4\xi_1}k^2 + \frac{\chi_1}{2\xi_1}k + g - \\ & \Big[(\eta_1 + \eta_2 + \eta_3)(u_1 - u_2) + (-\eta_1 + \eta_2 + \eta_3)(u_3 - u_4) \\ & + (\eta_1 - \eta_2 + \eta_3)(u_5 - u_6) + +(\eta_1 + \eta_2 + \eta_3)(u_7 - u_8)\frac{|k|k}{2}\Big] \\ & + \Big[(\chi_1 + \chi_2 + \chi_3)(u_1 - u_2) + (-\chi_1 + \chi_2 + \chi_3)(u_3 - u_4). \\ & +(\chi_1 - \chi_2 + \chi_3)(u_5 - u_6) + (\chi_1 + \chi_2 - \chi_3)(u_7 - u_8)\Big]|k|, \end{aligned}$$

ξ_i, η_i, χ_i, $i = 1, 2, 3$, are the coordinates of the vectors ξ^T, η^T, χ^T respectively, defined in Vázquez (2002).

Proof. The proof of the lemma follows the same way of proof developed in [7], but now we consider some simplifications that can be done by the fact that matrices $B_i \in \mathbb{R}^{3\times 3}$, $i = \overline{1,4}$, have rank one. ■

Remark 2 *In [7] were obtained the expressions for the calculation of the numbers* $\pi_1(A,(B_i)_{i\in\overline{1,4}})$, $\pi_2(A,(B_i)_{i\in\overline{1,4}})$ *and* $\pi_3(A,(B_i)_{i\in\overline{1,4}})$. *Now these expressions are more simple and we may see this fact by direct comparison.*

Next we show some examples and calculate the number $r^*(A,(B_i)_{i\in\overline{1,4}})$ considering the same perturbation matrices $B_i \in \mathbb{R}^{3\times 3}$, $i = \overline{1,4}$, for all cases.

Example 1

$$A = \begin{pmatrix} -1 & -1 & 1 \\ 3 & -1 & 3 \\ -2 & 1 & -4 \end{pmatrix}, b = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \nu^0 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} .$$

$\pi_1(A,(B_i)_{i\in\overline{1,4}}) = 6$, $\pi_2(A,(B_i)_{i\in\overline{1,4}}) = 0.42857$ *and* $\pi_3(A,(B_i)_{i\in\overline{1,4}}) = 2$, *then* $r^*(A,(B_i)_{i\in\overline{1,4}}) = 0.42857$.

Example 2

$$A = \begin{pmatrix} -20 & 48 & -1 \\ 2 & -10 & 1 \\ 1 & 20 & -4 \end{pmatrix}, b = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \nu^0 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}.$$

$\pi_1(A,(B_i)_{i\in\overline{1,4}}) = 34$, $\pi_2(A,(B_i)_{i\in\overline{1,4}}) = 0.20455$ *and* $\pi_3(A,(B_i)_{i\in\overline{1,4}}) = 8.80305$, *then* $r^*(A,(B_i)_{i\in\overline{1,4}}) = 0.20455$.

Example 3

$$A = \begin{pmatrix} -20 & 1 & -1 \\ 1 & -10 & 1 \\ 2 & 3 & -40 \end{pmatrix}, b = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \nu^0 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}.$$

$\pi_1(A,(B_i)_{i\in\overline{1,4}}) = 70$, $\pi_2(A,(B_i)_{i\in\overline{1,4}}) = 15.7163$ *and* $\pi_3(A,(B_i)_{i\in\overline{1,4}}) = 23.8605$, *then* $r^*(A,(B_i)_{i\in\overline{1,4}}) = 15.7163$.

3 Relationship between the number $r^*(A,(B_i)_{i\in\overline{1,4}})$ and the magnitude of the perturbation

Let us consider the system of differential equations

$$\dot{x} = (A + P)x,$$

where $A \in \mathbb{k}^{n\times n}$ is stable and $P \in \mathbb{k}^{n\times n}$ belongs to a certain class of perturbations; $\mathbb{k} = \mathbb{R}$ or $\mathbb{k} = \mathbb{C}$. If the matrix P is the null matrix, then we have a stable system of ordinary differential equations, because we are considering that the matrix A has all eigenvalues with negative real part.

In this section we find out a relationship between the number $r^*(A,(B_i)_{i\in\overline{1,4}})$ and the magnitude of the perturbation defined by the set considered in the first section.

Definition 5 *For the considered perturbation, we define its magnitude as* $m = \max\left\{\|B_i\|, i = \overline{1,4}\right\}$*, where* $\|\cdot\|$ *is any norm defined for matrices.*

The used norms in order to analyze the magnitude of the perturbations are the usual ones for matrices:

Infinite Norm, $\|M\|_\infty = \max_{i=\overline{1,n}} \sum_{j=1}^{N} |a_{i,j}|$.

Unit Norm, $\|M\|_1 = \max_{j=\overline{1,n}}, \sum_{i=1}^{N} |a_{i,j}|$.

Frobenius Norm, $\|M\|_{fro} = \sqrt[2]{\sum_{i=1}^{N}\sum_{j=1}^{N} a_{i,j}^2}$.

In the next lemma we show that the magnitude of the considered perturbation is precisely the norm of any of matrices $B_i \in \mathbb{R}^{3\times 3}$, $i = \overline{1,4}$.

Lemma 6 *Let be* $B_i \in \mathbb{R}^{3\times 3}$*,* $i = \overline{1,4}$*, the matrices defined in the first section, then the magnitude of the perturbation defined by these matrices is expressed by the equalities:*

$$\begin{aligned} m_\infty &= |b_1 v_1^0| + |b_1 v_2^0| + |b_1 v_3^0|, \\ m_1 &= \max\left\{|b_1 v_1^0|, |b_1 v_2^0|, |b_1 v_3^0|\right\}, \\ m_{fro} &= \sqrt[2]{(b_1 v_1^0)^2 + (b_1 v_2^0)^2 + (b_1 v_3^0)^2}, \end{aligned}$$

when the infinite norm, the unit norm or the Frobenius norm are considered respectively.

Proof. The matrices $B_i \in \mathbb{R}^{3\times 3}$, $i = \overline{1,4}$, are of rank one, then they may be transformed to a step form. The obtained matrices will have only one not null row (or column). Without loss of generality, let consider that the transformed matrices have not null only the first row. By direct calculation of the magnitude of the perturbation, using for example, the infinite norm, we obtain: $m_\infty = \max\{\|B_i\|, i = \overline{1,4}\}$, then will be

$$\|B_{k(k=\overline{1,4})}\|_\infty = \max_{i=\overline{1,3}} \sum_{j-1}^{3} |b_{ij}^{(k)}|.$$

From these expressions follow

$$\|B_{k(k=\overline{1,4})}\|_\infty = \max\{|b_{11}^{(k)}| + |b_{12}^{(k)}| + |b_{13}^{(k)}|\}.$$

By substitution in formulae for m_∞ we have

$$m_\infty = \max_{k=\overline{1,4}}\{\max\{|b_{11}^{(k)}| + |b_{12}^{(k)}| + |b_{13}^{(k)}|\}\},$$

that can be written as

$$m_\infty = \max_{k=\overline{1,4}}\{|b_{11}^{(k)}| + |b_{12}^{(k)}| + |b_{13}^{(k)}|\}.$$

But the values b_{1j}, $j = \overline{1,3}$ are different only by the sign for each $k = \overline{1,4}$, then their absolute values will be equals and the sum will be the same for all k , therefore $m_\infty = |b_{11}|+|b_{12}|+|b_{13}|$, i.e., $m_\infty = |b_1 v_1^0|+|b_1 v_2^0|+|b_1 v_3^0|$. In the same way we can proof statement for the other norms. ■

Example 4 *Determination of the magnitude of the perturbation.*

Taking the same perturbation considered in Example 1, where

$$b = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \nu^0 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix},$$

we obtain the results nthat appear in Table 1.

We observe that the magnitude of the perturbation can be calculated taking into account any of the matrices $B_i \in \mathbb{R}^{3\times 3}$, $i = \overline{1,4}$.

Example 5 *In this case we show how the magnitude of the perturbation affects the number* $r^*(A, (B_i)_{i\in\overline{1,4}})$.

	B_1	B_2	B_3	B_4
m_∞	4	4	4	4
m_1	2	2	2	2
m_{fro}	2.4495	2.4495	2.4495	2.4495

Table 1: Norms of the perturbation for different matrix norms in example 4.

We take the matrix

$$A = \begin{pmatrix} -1 & -1 & 1 \\ 3 & -1 & 3 \\ -2 & 1 & -4 \end{pmatrix}$$

of Example 1, and the respective perturbation for which $r^* = 0.42857$ *and* $\|B_i\| = 0.24495$. *In order to find the magnitude of the perturbation we use the Frobenius norm. Varying the magnitude of the perturbation we obtain:*

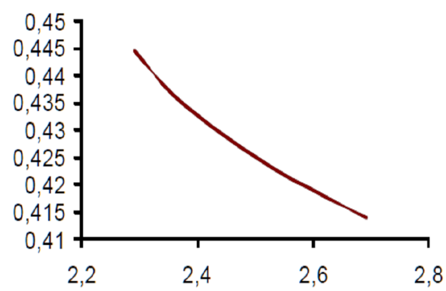

Figure 1: Relationship bettween the norm of the disturbance (horizontal axis) and the the number r^* (vertical axis), using the Frobenius matrix norm.

We see in Figure 1 that the number $r^*(A, (B_i)_{i\in\overline{1,4}})$ *decreases when the magnitude of the perturbation increases.*

In order to have an idea about the behavior of the stability properties for the studied polytope, we can do numerical tests for the variation of the system stability when the magnitude of the perturbation changes for $\overline{r} < r^*(A, (B_i)_{i\in\overline{1,4}})$ *fixed. Now we present a pair of examples in which we use the vectors*

$$b = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{ and } \nu^0 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}.$$

In order to varying the perturbation norm, we change the first coordinate of the vector ν^0.

Example 6 *For the matrix* $A = \begin{pmatrix} -1 & -1 & 1 \\ 3 & -1 & 3 \\ -2 & 1 & -4 \end{pmatrix}$, $r^*(A,(B_i)_{i\in\overline{1,4}}) = 0.42857$. *Taking* $\overline{r} = 0.428$ *follow results presented in Table 2.*

$\|B\|_\infty$	$\|B\|_1$	$\|B\|_{fro}$	Greatest eigenvalue	Stability
4.1	2	2.492	0.0049988	Not stable
4.019	2	2.4573	0.000018572	Not stable
4.018	2	2.4569	-0.000041636	Stable
4.005	2	2.4515	-0.00082157	Stable
4	2	2.4495	-0.0011202	Stable
3.95	2	2.4295	-0.0040651	Stable
3.8	2	2.3749	-0.012471	Stable

Table 2: Performance of the stability of the polytope in Example 6.

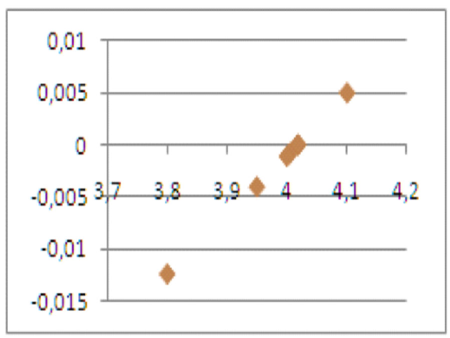

Figure 2: Relationship between the magnitude of the disturbance (horizontal axis) and the largest eigenvalue of the perturbation (vertical axis) in Example 6.

Example 7 *For the matrix* $A = \begin{pmatrix} -20 & 1 & -1 \\ 1 & -10 & 1 \\ 2 & 3 & -40 \end{pmatrix}$, $r^*(A,(B_i)_{i\in\overline{1,4}}) = 15.7163$. *Taking* $\overline{r} = 15.715$ *follow results presented in table 3.*

In the Tables and Figures 2 and 3 we observe that, for the given arbitrary stable matrix $A \in \mathbb{R}^{3\times 3}$ and matrices $B_i \in \mathbb{R}^{3\times 3}$, $i = \overline{1,4}$, and

$\|B\|_\infty$	$\|B\|_1$	$\|B\|_{fro}$	Greatest eigenvalue	Stability
4.1	2	2.492	1.1677	Not stable
4.05	2	2.4703	0.5752	Not stable
4.0002	2	2.4496	0.011065	Not stable
4.0001	2	2.44953	-0.000029336	Stable
4	2	2.4495	-0.0011651	Stable
3.95	2	2.4295	-0.56003	Stable
3.8	2	2.3749	-2.1182	Stable

Table 3: Performance of the stability of the polytope in Example 7.

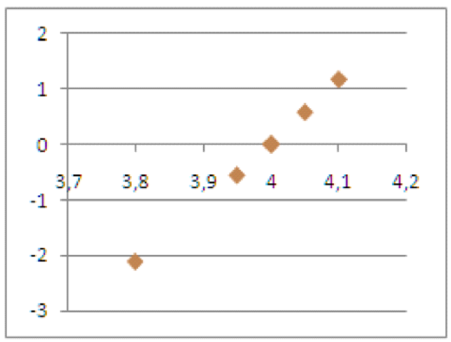

Figure 3: Relationship between the magnitude of the disturbance (horizontal axis) and the largest eigenvalue of the perturbation (vertical axis) in Example 7.

perturbation with considered characteristics, the number $r^*(A,(B_i)_{i\in\overline{1,4}})$ is the stability radius (see, for example, Hinrichsen & Pritchard (1988), [2]) for the polytope defined by those matrices. Taking a fixed positive number $\overline{r}$, so that $\overline{r} < r^*(A,(B_i)_{i\in\overline{1,4}})$, and varying the perturbation norm, we see that the stability of the polytope is conserved for small increases of the perturbation norm, i.e., taking the number $\overline{r}$, so that $\overline{r}+\varepsilon = r^*$, we can take a perturbation norm $\|B_i\|^*$, so that $\|B_i\|^* - \delta = \|B_i\|$ and the polytope conserves the stability.

4 Conclusion

In this work we simplify the method presented in [7] for the calculation of the number $r^*(A,(B_i)_{i\in\overline{1,4}})$, which characterizes the stability properties of the studied polytope. Also we calculate the norm of the considered perturbation and we determine the relationship between the numbers $r^*(A,(B_i)_{i\in\overline{1,4}})$ and $m_{\|\cdot\|}$. The numerical tests shown that the behavior of the polytope stability, with $\overline{r} < r^*(A,(B_i)_{i\in\overline{1,4}})$ fixed, is the expected one respect to the variation of the magnitude of the perturbation. So, we think in the possibility to extend the study of the stability properties of the considered polytope to the linear time depending case. In the examples 1, 2 and 3, we see that the equality $r^*(A,(B_i)_{i\in\overline{1,4}}) = \pi_2(A,(B_i)_{i\in\overline{1,4}})$ holds. Is it general for the considered perturbation?

References

[1] Galeev, E.; Tijomirov, V. (1991) *Breve Curso de la Teoría de Problemas Extremales*. Editorial Mir, Moscú.

[2] Hinrichsen, D.; Pritchard, A.J. (1988) "A robustness measure for linear systems under structured real parameter perturbations", Report No. 184, Institut für Dynamische Systeme, Universität Bremen.

[3] Kharitonov, V.L. (1978) "Asymptotic stability of an equilibrium position of a family of systems of linear differential equations", *Diffrentsial'nye Uravneniya* **14**(11).

[4] Mederos, O.; Grau, R.; Hing, R.; González, L.A. (1987) *Ecuaciones Diferenciales Ordinarias*. Editorial Pueblo y Educación, Cuba.

[5] Nicado, M. (1998) *Estudio del Comportamiento de Sistemas de Control Automático de Tercer Orden con Perturbaciones Estacionarias y no Estacionarias*. Tesis de Doctorado, Universidad Central de Las Villas, Villa Clara, Cuba.

[6] Loan, C. van (1985) "How near is a stable matrix to an unstable matrix?", *Contemporary Math.* **47**.

[7] Vázquez, E. (2002) "On the stability of class of polytopes of third order square matrices and stability radius", *Revista Matemática: Teoría y Aplicaciones* **9**(1): FALTA.