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To cite this article: R D Fallas-Soto *et al* 2021 *J. Phys.: Conf. Ser.* **1723** 012026

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240th ECS Meeting ORLANDO, FL

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Abstract submission due: April 9

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Finding meaning in quadratic equations through a physical context: falling objects motion

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Abstract. This paper presents the results of the implementation of a design focused on the resignification of the parameters of the quadratic equation (parabolas) through the context of a physical phenomenon. This design was based on an epistemology of practices framed in principles of cognitive and social constructivism, particularly the socioepistemological theory of mathematics education, based also on the model of guided discovery. Based on the students' arguments, evidence of resignification is shown in the parameters of the quadratic equation in such a way that relationships between algebraic and graphical expressions were established. It was identified that the students' experiences, as well as the context, were determining elements for success in the resolution of the design.

1. Introduction

Traditionally, mathematical objects are studied in school mathematics without becoming involved with other disciplines. It is well known that studying mathematical objects in isolation from other disciplines tends to be non-functional [1], it is simply based on algorithms and methods that the student applies and accommodates discrepancies between concepts studied in school and properties or behaviours associated with phenomena of physics, chemistry, etc. So, a fundamental characteristic of school mathematics, especially at the post-secondary level, is that it serves other scientific fields and other practices [1]. In the same way, historical and cultural contexts are often not considered in the learning of mathematics, where scientific phenomena could be used to understand content. In textbooks in the educational system, problems are described as a series of exercises without the student acquiring senses and meanings [2]. Another aspect which should be strongly considered in mathematics education is the incorporation of technology. It is known that students can learn to use technology as a checking and transformational device, which makes it easier for them to expose their cognitive conflicts so that they might challenge their thinking, and ultimately correct it [3]. A creative and innovative use of learning devices can provide benefits in the learning process [4].



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The teaching of quadratic equations (QEs) has been approached in several manners: algebraically [5], geometrically [6], promoting the use of dynamic environments [7], etc. Traditional approaches to teaching the QE have led to significant obstacles in student understanding of quadratic function [8]. The main problems which arise when understanding QEs often occur due to the weakness in understanding their underlying concept [6]. This is particularly worrisome since the study of QEs is critical for students to connect linear functions with polynomial functions, while also allowing them to explore mathematical patterns and relationships [9]. Learning conditions in a lot of countries only emphasise the use of an algorithm or a formula [6], including the topic of QEs [10]; such is the case of Mexican education.

In this work, the physical phenomenon of falling objects is used to resignify the QE differently from how it is presented in mathematical scenarios in the educational system; falling of objects is used to show functional development in learning the QE as a mathematical object; thus the student associates this phenomenon with the graph of the quadratic function. This paper argues that the practice of prediction, which is traditionally associated with the domain of physics, is associated with the process of resignification of the QE.

2. Theoretical framework

Even though “constructivism” may imply a vague concept due to the variety of meanings, it is currently considered in many teaching institutions as the best method for teaching and learning, where constructivist teaching strategies have had a great effect for the student [11]. Two major types of constructivism are recognised: cognitive (based on the works by Piaget [12]) and social (based on the works by Vygotsky [13]). Even though these two types of constructivism are essentially different, the main concept is that ideas are constructed from experience, rather than instruction, so that they have a personal meaning for the students. Basically, in cognitive constructivism, ideas are constructed through a personal process, while in social constructivism they are constructed through social interaction, both with the teacher as with other students.

This work is based on the principles of cognitive and social constructivism, particularly on the socioepistemological theory of educational mathematics (STME), which conceives practices as comparison, estimation, prediction, among others, as generators of mathematical knowledge [14]. For example, by comparing states, it is possible to study the change and variation of a phenomenon to estimate or predict a future one, situations that gave rise to the mathematical objects today known from calculus. Mathematics must cease to have a utilitarian use [15] (methodical, memory, fictitious applications) to be functional to society, which implies that the construction of mathematical knowledge must be encouraged so students can transform his environment. In this direction, one way is to redirect the attention of the objects towards the practices of which they underlie [15]. Thus, it is necessary to study mathematics “from” the everyday and not “for” the everyday. It is in other scientific, technical, or popular domains where mathematics acquires sense and meaning [2]. Even the practices allow to resignify mathematical concepts. Thus, it can be considered that both the practices and the use of a real situational context are key parts in the structure of learning situation designs that enable the construction of functional knowledge, related to the most advanced aspect of deep understanding: translating, determining, interpreting, solving, and using [16].

A model purposely designed to improve students’ participation in constructing their own knowledge is guided discovery. In this model, the teacher only acts as a facilitator to set the course of learning, and the learning activities are designed to promote the students’ involvement in the learning process [4]. This model has been shown to provide a positive impact on the development of students’ critical and independent thinking, focusing on understanding. The students can also analyse themselves so that they can find the underlying concepts based on material or data provided by the instructor [4]. Guided forms of teaching are necessary for students to construct their own concepts and understanding of what is being taught [11].

3. Design and method

The objective of this work is to resignify the QE ($y = ax^2 + bx + c$) —as well as each of the coefficients of the analytical expression—from a design of the learning situation proposed from the graphing-modelling, leaning on the physical model of falling objects. The process of epistemological significance that takes place when the reference point is “the use” (in this case, the QE in specific situations), is understood as “resignification” [17]. Learning situations, from the socioepistemological approach, work with mathematical knowledge in a real situational context along with a pragmatic evolution.

Prediction plays a fundamental role in the learning of mathematics [18]; the questions of the situations (moments) were organised according to the practice of this activity [19]:

1. The activities are related to graphs and movement to study the changes and the simultaneity of the derivatives: distance, speed, acceleration.
2. The activities are related to the graphing-modelling to resignify the equation of the parabola and the linearity of the polynomial.
3. The prediction model where the previous moments are integrated is examined.

These moments are considered in the didactic engineering methodology [20] from the socioepistemological position [21]:

1. Preliminary analysis: the recognition of the teaching status of this content in the didactic dimension, the epistemology of calculus reported in [19] and the cognitive component of the population to which the learning situation will be applied.
2. *A priori* analysis: The three moments are considered, in addition to the didactic control variables (emphasis on the linearity of the polynomial, use of motion sensors and image processors, practices such as comparison and prediction guiding the elaboration of queries).
3. Post-analysis: Performing the staging, data analysis and results.

This study involved 5 students between the ages of 21 and 23 years, coursing their last year of Geophysical Engineering at the Mexican National Polytechnic Institute. The dynamic of the activity was initially individual, and every participant had sheets of paper and pencils. During the individual activities, the students received guidance from the facilitators, but knowledge was expected to be brought out of them from their own experience. After these activities, the students participated in social discussion and dialoguing between them and the facilitators.

For data collection, the Texas Instrument model CBR2/PWB/1L1/A motion sensor was used. This sensor was placed on a tripod at a height of 1.5m and pointed towards the ground, measuring the distance between an object and the sensor itself. The sensor was connected directly to a computer so the data could be processed and graphics with less environmental noise could be presented to the students on the screen. A design associated with the concept of parabola was developed and experimented for the resignification of its coefficients: a , b , c , in the expression $ax^2 + bx + c$ based on a graphing category and which was based on elements of the STME. The design that was staged was based on the principles of guided discovery models, and consisted of the following activities:

1. Make the graph of the distance of the object to the point from which it is dropped, with respect to time. Also, make graphs regarding the velocity and acceleration of the object.
2. From the graphs obtained by the sensor, give an explanation and interpretation of them.
 - a. How is the behaviour during the fall of the object?
 - b. Does the behaviour of the graph that you previously performed resemble the one obtained by the sensor?
3. From the obtained graphs of velocity and acceleration, give an explanation and interpretation of them.
 - a. Does the behaviour of the last graphs resemble those obtained by computer?

- b. Find an analytical expression for the graph of acceleration.
- c. Justify that the following is true: $a(t) = \frac{d^2s}{dt^2} = 9.8$, $v(t) = \frac{ds}{dt} = 9.8t + v_0$, $s(t) = 4.9t^2 + v_0t + s_0$.
4. Given the following graphs, describe how the object was launched in terms of the initial distance and velocity (find s_0 and v_0), and check your guesses with the aid of the sensor.
 - a. Determine the slope and intercept of the tangent line to the curve at $t = 0$ (see figure 1).
5. Given the following equations that describe the distance of a falling object from time, explain the physical behaviour associated with the analytic expression (particularly in terms of initial distance and velocity) and sketch the graph: $s(t) = 4.9t^2 + (-2)t + (1)$, $s(t) = 4.9t^2 + (2)t + (-1)$.
 - a. Sketch the graph of the following expressions: $f(x) = 4.9x^2 + (0)x + (0)$, $f(x) = 4.9x^2 + (3)x + (-1)$, $f(x) = 4.9x^2 + (-2)x + (2)$, $f(x) = 10x^2 + (0)x + (1)$, $f(x) = -10x^2 + (-2)x + (-1)$.

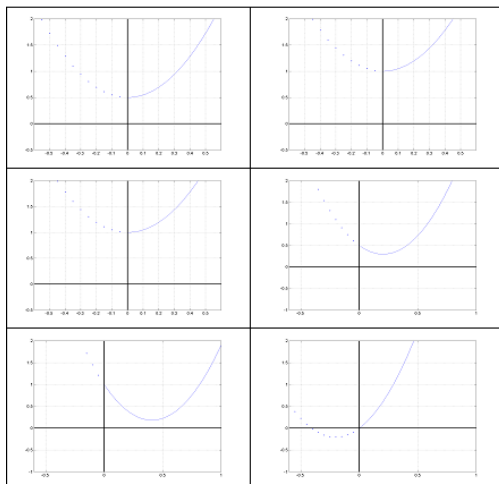


Figure 1. Graphs presented to students as part of Activity 4.

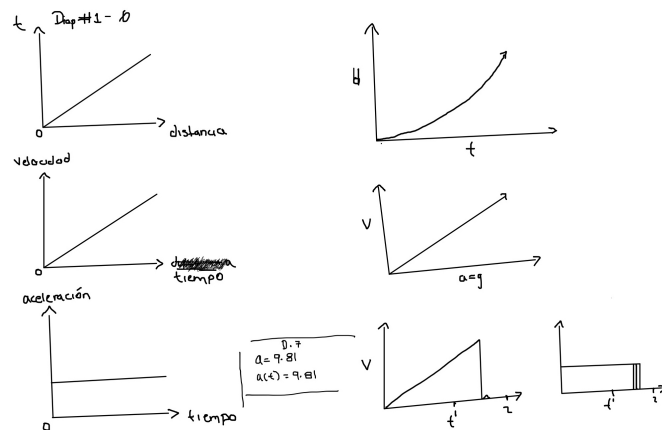


Figure 2. Students' responses to Activity 1.

4. Results

The situation and its context allow to build and give meaning to the reference system, where 0 m corresponds when the object is at the same height as the sensor, and 1.5 m when it touches the ground. Thus, the physical context of the learning situation gives meaning to the graphs obtained and their respective analytical expressions. This is evidenced in the sentences of the students such as: “the object was dropped from a height of...”, “here the object is already on the ground”, “here the object is not yet dropped...”, “this fragment of graph corresponds to errors in data collection”, “here the object bounced”, “this part corresponds to where the object was thrown”, etc. In these sentences, it is shown that the graph is related to the physical behaviour of throwing or dropping an object. Even students when interpreting the analytical expression make use of the context, e.g. expressions of the type $ax^2 + bx + c$ mention that a is related to acceleration, b to the velocity, and c to the distance, at which the object is dropped or thrown. It was observed that most of the students have a linear thinking regarding physical behaviours, since most of the students graphed a straight line to represent the fall of the object (figure 2).

The role of the students' experiences, as well as their daily life, allowed the establishment of conjectures for the graphing category that were requested in Activity 1, for example, they established that the behaviour of the fall of an object would be exponential, what they later

refuted. It is evident that students, when faced with graphs that present an upward concave curvature, tend to associate it with exponential and non-quadratic behaviour. This may mean that students assume behaviours with global and non-local characteristics, without verifying it through some type of strategy such as a numerical analysis.

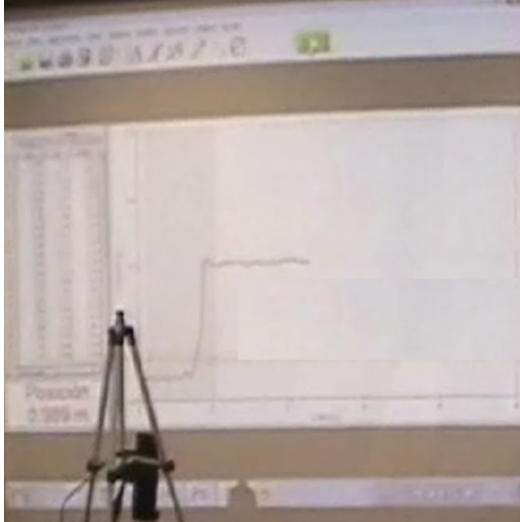


Figure 3. Sensor configuration for the development of activities. Particularly, a graph of the distance measured by the sensor when dropping an object is shown.

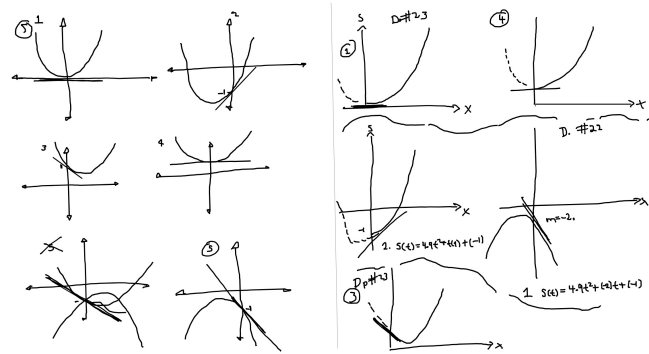


Figure 4. Graphs made by students in Activity 5.

The use of the sensor turned out to be a means that enabled the students' interaction with the mathematical object (parabola) through graphing. In addition, it enabled students to make guesses and modify their answers. E.g., when they associate an exponential behaviour of the graph generated by the sensor (figure 3), they compare it with the sensor-based graph when the same object is thrown upwards, thus agreeing that indeed the behaviour is parabolic. The meanings students gave to the behaviour of graphs contributed to their arguments to justify and convince themselves of the relationships between graphical and algebraic expressions of distance, velocity, and acceleration: $a(t) = \frac{d^2s}{dt^2} = 9.8$, $v(t) = \frac{ds}{dt} = 9.8t + v_0$, $s(t) = 4.9t^2 + v_0t + s_0$.

School experiences allowed students to establish that the previous algebraic expressions corresponded to the acceleration, velocity, and distance of an object, which is either dropped or thrown. Since they used their knowledge of physics to establish that the derivative of distance is velocity, and that the derivative of velocity is acceleration, they even used this argument to convince themselves algebraically, since they computed the respective derivatives, affirming that the first expression is true, since the acceleration in this class of phenomena is gravity (9.8 m/s^2).

It was observed that the graphing, together with the physical context, allowed the students to develop the argumentation of the linearity of the polynomial as a tool to obtain the graph of a QE. This is evidenced in their use of the graphing in Activity 5. In figure 4 it is important to observe how the students did not recur to tabulation or calculation of the vertex to plot the parabola, but instead used the linear part of the polynomial (the tangent lines to the parabola at $t = 0$). In addition, it was shown that this argument (linearity of the polynomial) made it possible to resignify the parameters of the QE. Since the students alluded that c represents the cut-off point with the y -axis, b represents the slope of the line tangent to that point and a represents whether it is concave up or down. This can be seen in the way they make their graphs (figure 4) and it is even made explicit when they are questioned about these parameters

and the relationship with their graphical representation.

5. Conclusions

The establishment of an epistemology of object and practices serves as the basis for designing situations that expand the possibilities of success for students to resignify their mathematical knowledge. It was possible to see that a design based on an epistemology allows the student to generate an argument (linearity of the polynomial) as a strategy to graph a parabola from its analytical expression without resorting to the use of traditional methods. This argument consists of establishing relationships between a polynomial function and the equation of the line, through determining a behaviour that tends to another when values close to zero are considered; with this, the student will reconstruct meanings to the relationships [19]. Likewise, the student realises how the use of a motion sensor along with the graphing category favours the argumentation of the linearity of the polynomial. Said formulated situation based on an epistemology of practices consisted of five activities in which the resignification of the parabola equation was favoured. That is, the students constructed meanings around the parameters that make up a general expression of a parabola. They also constructed arguments that supported predicting the graphical representation and behaviour of the analytical expression of the parabola, without the need to perform traditional methods, and, given the graph of a parabola, they are able to deduce the QE. In this situation, we drew from students' everyday knowledge to achieve mathematical knowledge and thus favour significant results. For this reason, the physical context played an important role for the argumentation made by the students.

Acknowledgments

This work was partially supported by Tecnológico Nacional de México / IT de Mérida project # 8285.20-P, as well as by projects 20200378, 20201040, 20202014, 20201606, 20200259, and EDI grant, by Secretaría de Investigación y Posgrado, Instituto Politécnico Nacional. Authors also thank the University of Costa Rica for the study grant that supported this research.

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